

第二节 电通量 高斯定律

1、C; 2、A; 3、B; 4、 $\pi R^2 E$

5、解：以 o 为球心，r 为半径作一同心球面作为高斯面，

$$r < R_1, \Phi = \oint \vec{E} \cdot d\vec{S} = \oint E \cdot dS = 0, E = 0$$

$$R_1 < r < R_2, \Phi = \oint \vec{E} \cdot d\vec{S} = \oint E \cdot dS = E \cdot 4\pi r^2 = \frac{Q_1}{\varepsilon_0}, E = \frac{Q_1}{4\pi\varepsilon_0 r^2}$$

$$r > R_2, \Phi = \oint \vec{E} \cdot d\vec{S} = \oint E \cdot dS = E \cdot 4\pi r^2 = \frac{Q_1 + Q_2}{\varepsilon_0}, E = \frac{Q_1 + Q_2}{4\pi\varepsilon_0 r^2}$$

6、解：以轴为中心，以 r 为半径，作高为 l 的同心圆柱面作为高斯面。

$$\text{由高斯定理 } \Phi = \oint \vec{E} \cdot d\vec{S} = \frac{Q_{in}}{\varepsilon_0} \text{ 得 } \Phi = \oint \vec{E} \cdot d\vec{S} = 2\pi r l E = \frac{Q_{in}}{\varepsilon_0}$$

$$\text{当 } r > R_2 \text{ 时, } Q_{in} = (\lambda_1 + \lambda_2)l, E = \frac{\lambda_1 + \lambda_2}{2\pi\varepsilon_0 r}$$

7、(1) 取同心球面为高斯面，由高斯定理求 E

$$r \leq R: E \cdot 4\pi r^2 = \frac{1}{\varepsilon_0} \int \rho dV = \frac{1}{\varepsilon_0} \int_0^r \frac{3Q}{\pi R^3} \left(1 - \frac{r'}{R}\right) 4\pi r'^2 \cdot dr'$$

$$E = \frac{\rho_0 r(4R - 3r)}{12\varepsilon_0 R} = \frac{Qr(4R - 3r)}{4\pi\varepsilon_0 R^4}$$

$$r > R: E \cdot 4\pi r^2 = \frac{1}{\varepsilon_0} \int \rho dV = \frac{1}{\varepsilon_0} \int_0^R \frac{3Q}{\pi R^3} \left(1 - \frac{r'}{R}\right) 4\pi r'^2 \cdot dr'$$

$$E = \frac{\rho_0 R^3}{12\varepsilon_0 r^2} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$(2) \frac{dE}{dr} = 0 \Rightarrow r = \frac{2}{3}R \text{ 时, } E = E_{\max} = \frac{\rho_0 R}{9\varepsilon_0} = \frac{Q}{3\pi\varepsilon_0 R^2}$$

8、解：以 x 轴为轴线，r 为半径，作一个圆柱面作为高斯面，圆柱面的上下底面距离原点 0 距离为 x。

$$\text{由高斯定理 } \Phi = \oint \vec{E} \cdot d\vec{S} = \frac{Q_{in}}{\varepsilon_0} \text{ 得 } \Phi = \oint \vec{E} \cdot d\vec{S} = 2\pi r^2 E = \frac{Q_{in}}{\varepsilon_0}$$

$$\text{当 } |x| < \frac{d}{2} \text{ 时, } Q_{in} = \pi r^2 \cdot 2x\rho, E = \frac{1}{2\pi r^2} \pi r^2 \cdot 2x\rho = \frac{x\rho}{\varepsilon_0}$$

$$\text{当 } |x| \geq \frac{d}{2} \text{ 时, } Q_{in} = \pi r^2 \cdot d\rho, E = \frac{1}{2\pi r^2} \pi r^2 d\rho = \frac{d\rho}{2\varepsilon_0}$$