

第三节 电势 电势能

1、C; 2、D; 3、 $\frac{3\sqrt{3}q}{2\pi\epsilon_0 a}$; 4、 $\frac{Q}{4\pi\epsilon_0 R^2}, 0; \frac{Q}{4\pi\epsilon_0 R}, \frac{Q}{4\pi\epsilon_0 r_2}$; 5、>, >;

6、解法一：由高斯定理可知，空间各处电场分布为：

$$r < R_1, E = 0$$

$$R_1 < r < R_2, E = \frac{Q_1}{4\pi\epsilon_0 r}$$

$$r > R_2, E = \frac{Q_1 + Q_2}{4\pi\epsilon_0 r}$$

r 处的电势和该处电场的关系为： $V = \int_r^\infty E dr$

$$r < R_1, V = \int_r^{R_1} E dr + \int_{R_1}^{R_2} E dr + \int_{R_2}^\infty E dr = \frac{Q_1}{4\pi\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) + \frac{Q_1 + Q_2}{4\pi\epsilon_0 R_2} = \frac{Q_1}{4\pi\epsilon_0 R_1} + \frac{Q_2}{4\pi\epsilon_0 R_2}$$

$$R_1 < r < R_2, V = \int_r^{R_2} E dr + \int_{R_2}^\infty E dr = \frac{Q_1}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{R_2} \right) + \frac{Q_1 + Q_2}{4\pi\epsilon_0 R_2} = \frac{1}{4\pi\epsilon_0} \left(\frac{Q_1}{r} + \frac{Q_2}{R_2} \right)$$

$$r > R_2, V = \int_r^\infty E dr = \int_{R_2}^\infty \frac{Q_1 + Q_2}{4\pi\epsilon_0 r^2} dr = \frac{Q_1 + Q_2}{4\pi\epsilon_0 r}$$

解法二：采用电势叠加法（可参照课件）

7、解：法一：以场源所在位置为原点，水平向右为正方向，建立直角坐标系，设想将单位正电荷从 M 点移到 P 点，电场力做功为

$$W_{MP} = \varphi_M = \frac{1}{4\pi\epsilon_0} \int_{2a}^a \frac{q}{x^2} dx = -\frac{q}{8\pi\epsilon_0 a}$$

法二：以无限远的位置为零势能点， $V_P = \frac{1}{4\pi\epsilon_0} \frac{q}{2a}, V_M = \frac{1}{4\pi\epsilon_0} \frac{q}{a}$

以 P 点为零势能点， $V_{MP} = V_M - V_P = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{2a} - \frac{q}{a} \right) = -\frac{1}{8\pi\epsilon_0} \frac{q}{a}$

8、解：在圆环上取一微元 dl，所带电量为 $dq = \lambda dl$ ，

$$dq \text{ 在 P 点处产生的电势为 } dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \frac{\lambda dl}{\sqrt{a^2 + R^2}},$$

$$\text{电势为 } V = \int dV = \int_0^{2\pi R} \frac{1}{4\pi\epsilon_0} \frac{\lambda dl}{\sqrt{a^2 + R^2}} = \frac{1}{4\pi\epsilon_0} \frac{2\pi R\lambda}{\sqrt{a^2 + R^2}} = \frac{1}{2\epsilon_0} \frac{R\lambda}{\sqrt{a^2 + R^2}}$$

电场力所做的功为

$$W = -\Delta E_p = q(V_a - V_b) = q\left(\frac{1}{2\epsilon_0} \frac{R\lambda}{\sqrt{a^2 + R^2}} - \frac{1}{2\epsilon_0} \frac{R\lambda}{\sqrt{b^2 + R^2}}\right) = \frac{qR\lambda}{2\epsilon_0} \left(\frac{1}{\sqrt{a^2 + R^2}} - \frac{1}{\sqrt{b^2 + R^2}}\right)$$