

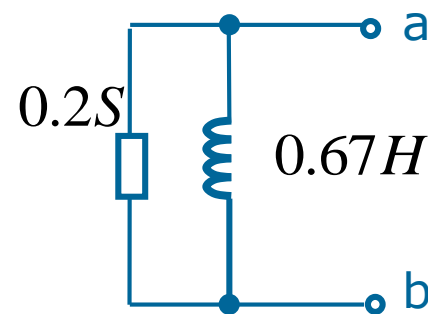
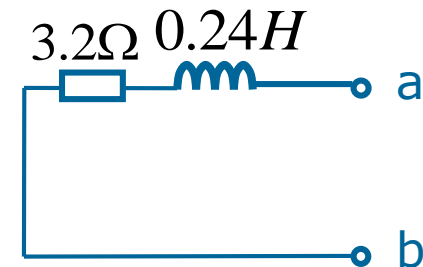
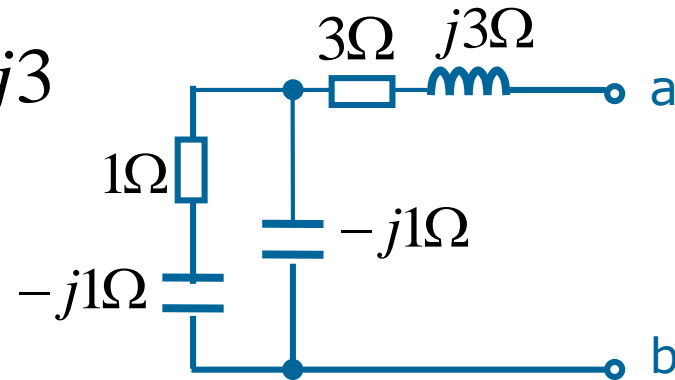
7-13 试求题图7-13所示电路的输入阻抗和导纳，以及该电路的最简串联等效电路和并联等效电路 $\omega = 10 \text{ rad/s}$ 。

解:
$$Z_{ab} = \frac{(1-j1)(-j1)}{(1-j1) + (-j1)} + 3 + j3$$

$$= \frac{-1-j1}{1-j2} + 3 + j3$$

$$= \frac{1-j3}{5} + 3 + j3 = 3.2 + j2.4 \Omega$$

$$Y_{ab} = \frac{1}{Z_{ab}} = \frac{1}{\frac{16}{5} + j\frac{12}{5}} = 0.2 - j0.15 \text{ S}$$



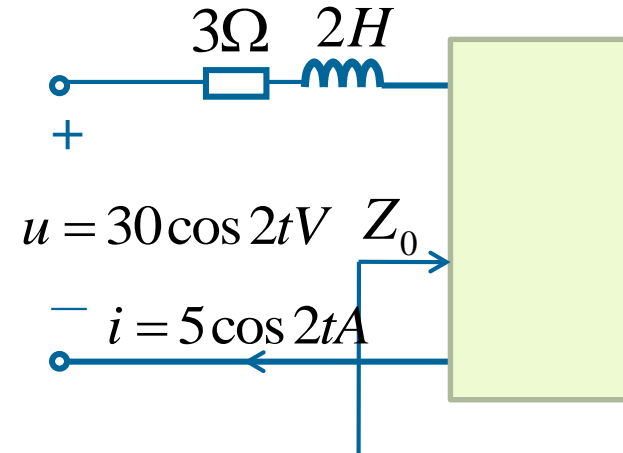
7-14 电路如题图7-14所示，试确定方框内最简串联等效电路的元件值。

解： $\dot{U}_m = 30\angle 0^\circ \text{V}$, $\dot{I}_m = 5\angle 0^\circ \text{A}$

$$\therefore Z = \frac{30\angle 0^\circ}{5\angle 0^\circ} = 6\Omega$$

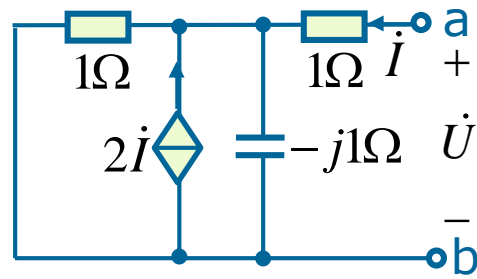
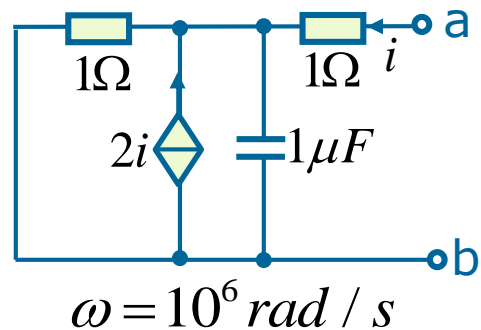
$$\therefore Z_0 = Z - 3 - j2 \times 2 = 3 - j4\Omega$$

$$\therefore R_0 = 3\Omega, C = \frac{1}{2 \times 4} = 0.125\text{F}$$



7-15(a) 试求题图7-15所示各二端网络的输入阻抗。

解:



$$\dot{U} = 1 \cdot \dot{I} + [1 / (-j1)] (\dot{I} + 2\dot{I})$$

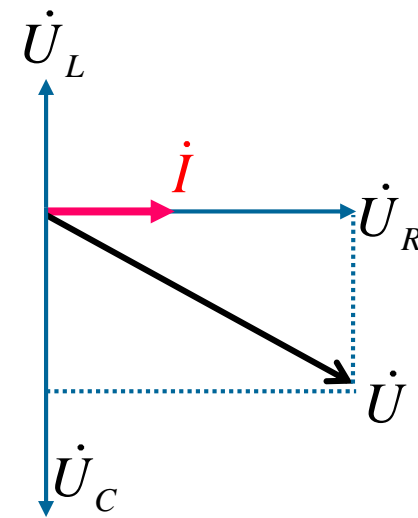
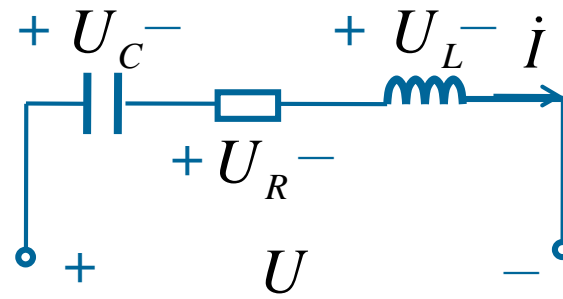
$$\dot{U} = \dot{I} + \frac{-j1}{1-j1} \cdot 3\dot{I} = \frac{1-j4}{1-j1} \dot{I} = (2.5 - j1.5) \dot{I}$$

$$\therefore Z_i = (2.5 - j1.5) \Omega$$

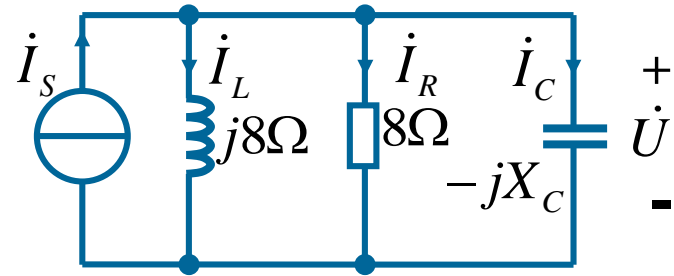
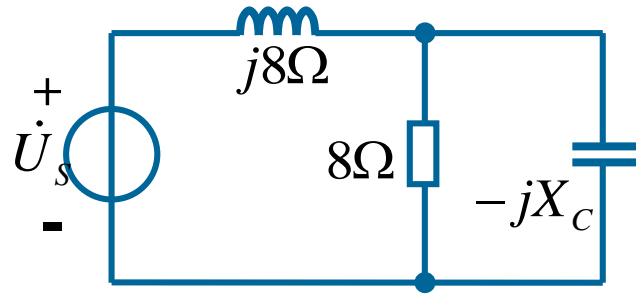
7-16 在题图7-16所示电路中, 已知 $U_C = 15V, U_L = 12V,$
 $U_R = 4V,$ 求电压 U 为多少?

解:
$$U = \sqrt{U_R^2 + (U_L - U_C)^2}$$

$$= \sqrt{4^2 + (12 - 15)^2} = 5V$$



7-27 二端网络如题图7-27所示，已知 $\dot{U}_S = 50\angle 0^\circ \text{V}$ ，电源提供的平均功率为 312.5W ，试求 X_C 的数值。



解：将电路等效为诺顿模型，并设各支路电流和电压如相量模型图所示，其中：

$$\dot{I}_S = \frac{\dot{U}_S}{j8} = 6.25\angle -90^\circ \text{A}$$

$$P = I_R^2 R = 312.5 \Rightarrow I_R = 6.25\text{A}$$

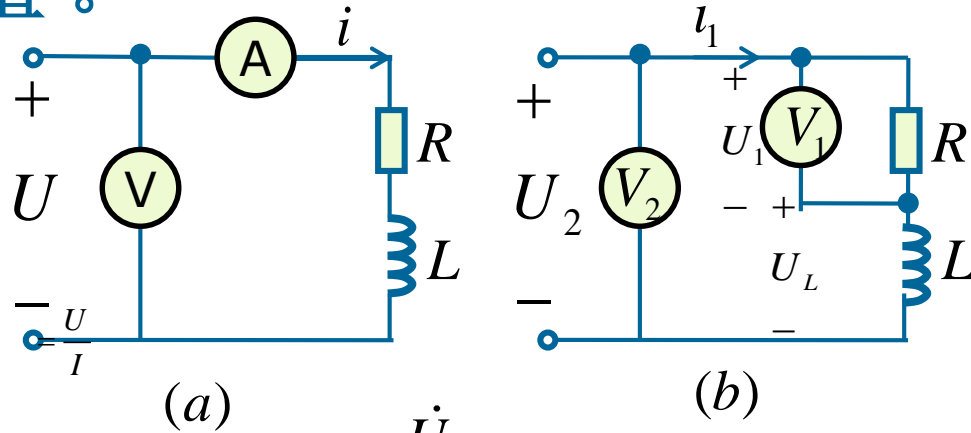
$$\because \dot{I}_S = \dot{I}_R + \dot{I}_C + \dot{I}_L \quad \text{即} \quad I_S = \sqrt{(I_C - I_L)^2 + I_R^2} \quad \text{且} \quad \because I_R = I_S$$

$$\therefore I_C = I_L \quad \text{而} \quad I_C = \frac{U}{X_C}, I_L = \frac{U}{8} \quad \therefore X_C = 8\Omega$$

7-18 RL串联电路，在题图7-18(a)直流情况下，电流表的读数为50mA，电压表的读数为6V。在 $f = 10^3 \text{ Hz}$ 交流情况下，电压表 V_1 读数为6V， V_2 读数为10V，如图(b)所示。试求R、L的值。

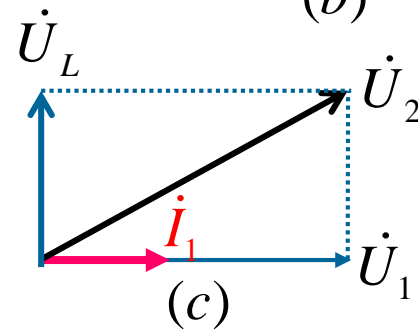
解：直流时电感相当于短路，则：

$$R = \frac{U}{I} = \frac{6}{50 \times 10^{-3}} = 120 \Omega$$



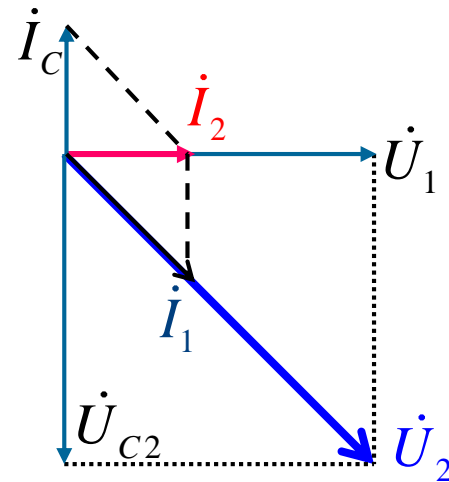
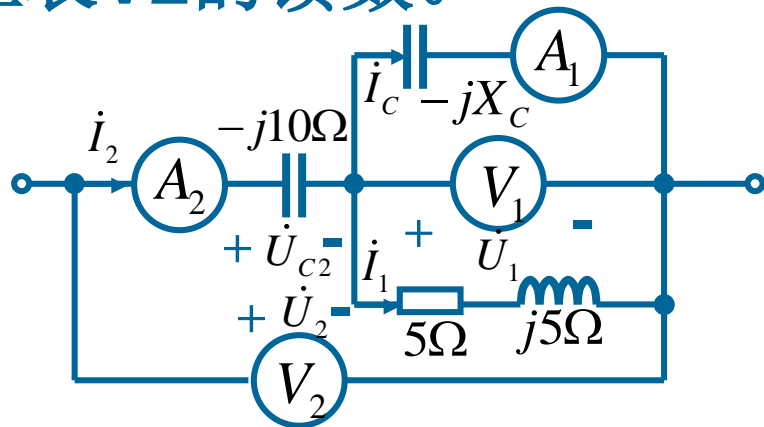
交流时相量图如图(c)，则：

$$U_L = \sqrt{U_2^2 - U_1^2} = 8 \text{ V}$$



$$\therefore \frac{U_1}{R} = \frac{U_L}{\omega L} \quad \therefore L = \frac{U_L R}{\omega U_1} = \frac{8 \times 120}{2\pi \times 10^3 \times 6} = 25.5 \text{ mH}$$

7-19 题图7-19所示电路，已知电流表A1的读数为10A，电压表V1的读数为100V；试画相量图求电流表A2和电压表V2的读数。



解：设各电压、电流如图，且设 \dot{U}_1 为参考向量，则：

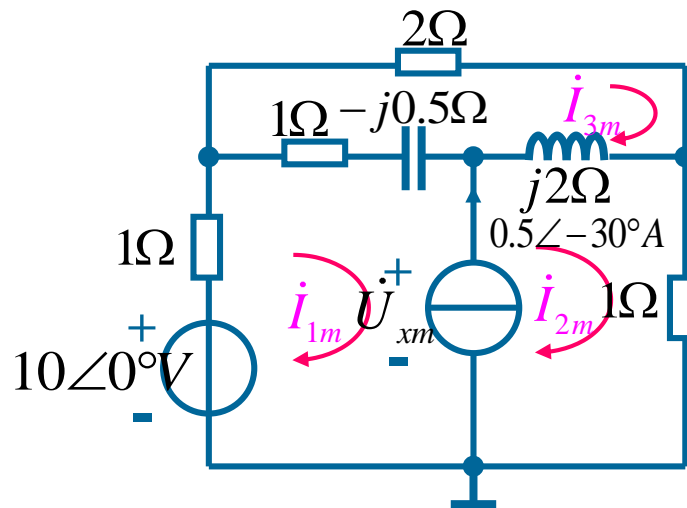
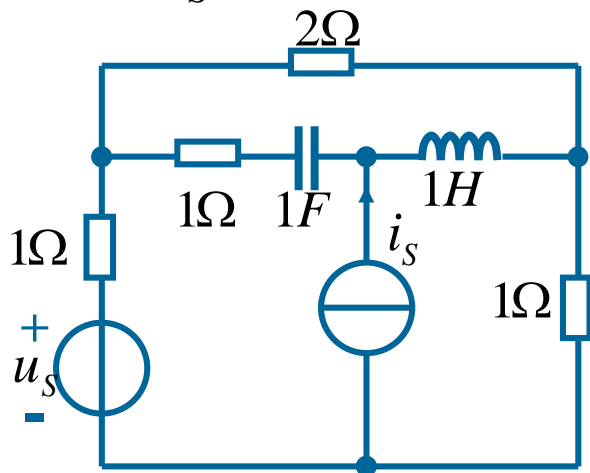
$$\dot{U}_1 = 100 \angle 0^\circ \quad \therefore \dot{I}_C = \frac{\dot{U}_1}{-jX_C} = \frac{U_1 \angle 0^\circ}{X_C \angle -90^\circ} = 10 \angle 90^\circ \text{ A}$$

$$\dot{I}_1 = \frac{\dot{U}_1}{5 + j5} = \frac{100 \angle 0^\circ}{5\sqrt{2} \angle 45^\circ} = 10\sqrt{2} \angle -45^\circ \text{ A}$$

故由相量图可得： $\therefore \dot{I}_2 = \dot{I}_1 + \dot{I}_C = 10 \angle 0^\circ \text{ A}$ 由相量图：

$$\dot{U}_{C2} = (-j10)\dot{I}_2 = 100 \angle -90^\circ \text{ V} \quad \dot{U}_2 = \dot{U}_{C2} + \dot{U}_1 = 100\sqrt{2} \angle -45^\circ$$

7-22(b) 试分别列写下列电路的网孔方程和节点方程，各图中 $u_s = 10 \cos 2tV$ ， $i_s = 0.5 \cos(2t - 30^\circ)A$ 。

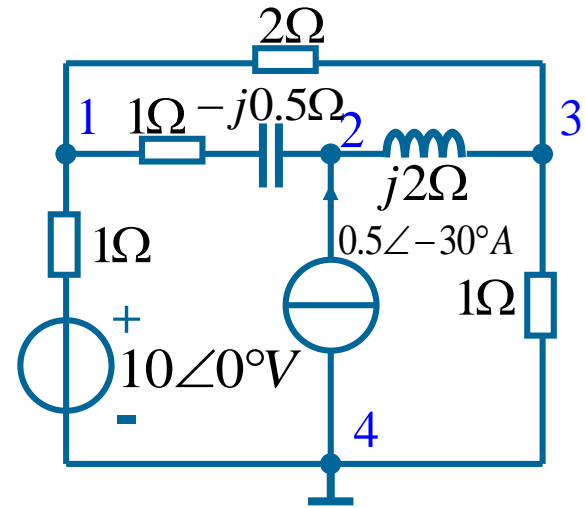


解：(1) 列网孔方程：

$$\begin{cases} (2 - j0.5)\dot{I}_{1m} - (1 - j0.5)\dot{I}_{3m} = 10\angle 0^\circ - \dot{U}_{xm} \\ (1 + j2)\dot{I}_{2m} - j2\dot{I}_{3m} = \dot{U}_{xm} \\ -(1 - j0.5)\dot{I}_{1m} - j2\dot{I}_{2m} + (3 + j1.5)\dot{I}_{3m} = 0 \\ \dot{I}_{2m} - \dot{I}_{1m} = 0.5\angle -30^\circ \end{cases}$$

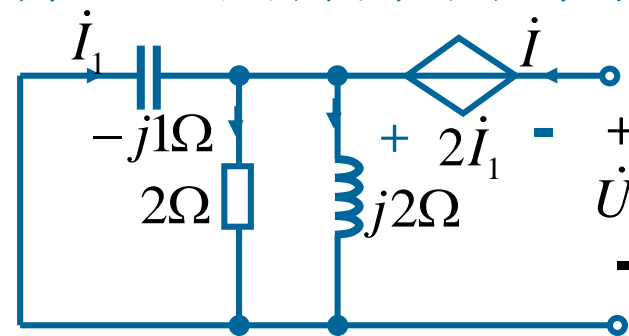
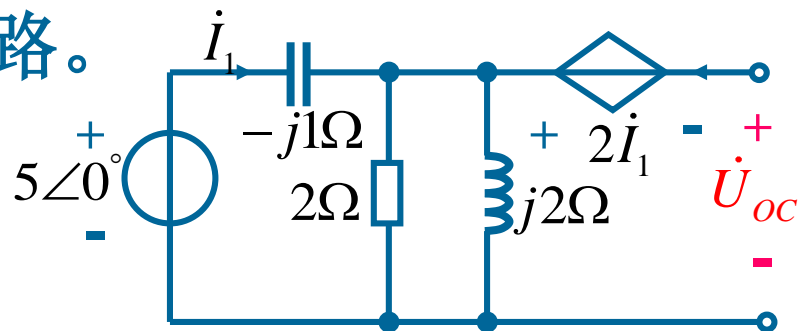
(2) 列节点方程:

$$\begin{cases} \left(1 + \frac{1}{2} + \frac{1}{1-j0.5}\right) \dot{U}_{1m} - \frac{1}{1-j0.5} \dot{U}_{2m} - \frac{1}{2} \dot{U}_{3m} = \frac{10\angle 0^\circ}{1} \\ -\frac{1}{1-j0.5} \dot{U}_{1m} + \left(\frac{1}{2j} + \frac{1}{1-j0.5}\right) \dot{U}_{2m} - \frac{1}{j2} \dot{U}_{3m} = 0.5\angle -30^\circ \\ -\frac{1}{2} \dot{U}_{1m} - \frac{1}{j2} \dot{U}_{2m} + \left(\frac{3}{2} + \frac{1}{j2}\right) \dot{U}_{3m} = 0 \end{cases}$$



$$\therefore \begin{cases} \left(\frac{3}{2} + \frac{1}{1-j0.5}\right) \dot{U}_{1m} - \frac{1}{1-j0.5} \dot{U}_{2m} - \frac{1}{2} \dot{U}_{3m} = 10\angle 0^\circ \\ -\frac{1}{1-j0.5} \dot{U}_{1m} + \left(-j\frac{1}{2} + \frac{1}{1-j0.5}\right) \dot{U}_{2m} + j\frac{1}{2} \dot{U}_{3m} = 0.5\angle -30^\circ \\ -\frac{1}{2} \dot{U}_{1m} + j\frac{1}{2} \dot{U}_{2m} + \left(\frac{3}{2} - j\frac{1}{2}\right) \dot{U}_{3m} = 0 \end{cases}$$

7-23(b) 试求题图7-23所示有源二端网络的戴维南等效电路。



解: (1) 求开路电压 \dot{U}_{oc} :

$$\begin{cases} \dot{U}_{oc} = -2\dot{I}_1 - (-j1)\dot{I}_1 + 5\angle 0^\circ \\ \dot{I}_1 = \frac{5\angle 0^\circ}{-j1 + (2 // j2)} = 5\angle 0^\circ \end{cases}$$

$$\therefore \dot{U}_{oc} = -5 + j5 = 5\sqrt{2}\angle 135^\circ$$

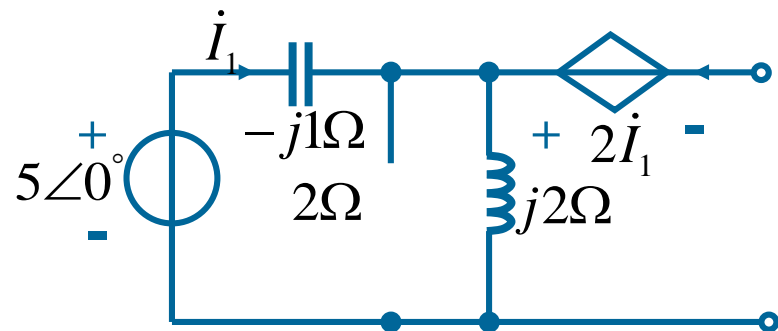
(2) 求等效阻抗 Z_0 :

$$\begin{cases} \dot{U} = -2\dot{I}_1 - (-j\dot{I}_1) \\ \dot{I} = -\dot{I}_1 + \frac{(-j1)(-\dot{I}_1)}{2} + \frac{(-j1)(-\dot{I}_1)}{j2} \end{cases}$$

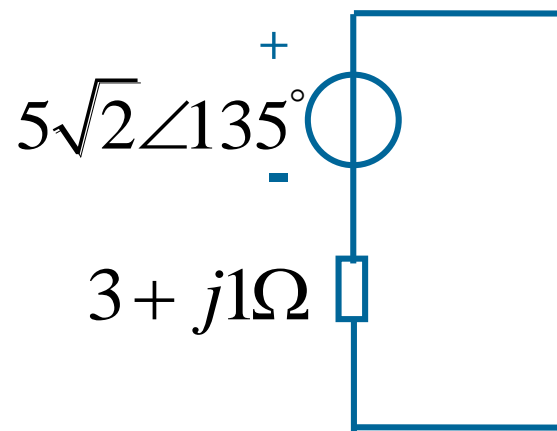
$$\text{或: } \begin{cases} \dot{U} = -2\dot{I}_1 - (-j\dot{I}_1) \\ \dot{I}_1 = -\frac{\frac{1}{-j1}}{\frac{1}{-j1} + \frac{1}{2} + \frac{1}{j2}} \dot{I} \text{ (分流公式)} \end{cases}$$

$$\text{可得: } \dot{U} = (3 + j1)\dot{I}$$

$$\therefore Z_0 = (3 + j1)\Omega$$



戴维南等效电路图:



说明: 若给的图是时域的, 则等效戴维南电路图也必须是时域的, 即: 要将 $Z_0 = (3 + j1)$ 转换成一电阻串联电感。

7-25 (2) 已知关联参考方向下的无源二端网络的端口电压 $u(t)$ 和电流 $i(t)$ 分别为 $u(t) = 10 \cos(100t + 70^\circ) \text{V}$ 和 $i(t) = 2 \cos(100t + 40^\circ) \text{A}$, 试求各种情况下的 P 、 Q 和 S 。

解: 先将各量写成相量形式:

$$\dot{U} = 5\sqrt{2} \angle 70^\circ \text{V}, \quad \dot{I} = \sqrt{2} \angle 40^\circ \text{A}$$

$$P = UI \cos \theta_z = 5\sqrt{2} \times \sqrt{2} \cos 30^\circ = 5\sqrt{3} \text{W}$$

$$Q = UI \sin \theta_z = 5\sqrt{2} \times \sqrt{2} \sin 30^\circ = 5 \text{Var}$$

$$S = UI = 10 \text{VA}$$

另解: $\tilde{S} = \dot{U} \dot{I}^* = 5\sqrt{2} \angle 70^\circ \times \sqrt{2} \angle -40^\circ$

$$= 10 \angle 30^\circ = 5\sqrt{3} + j5$$

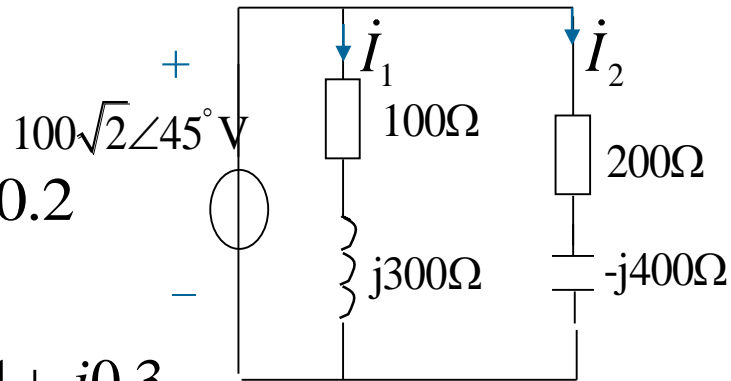
$$\therefore P = 5\sqrt{3} \text{W}, \quad Q = 5 \text{Var}, \quad S = 10 \text{VA}$$

7-26 试求题图7-26所示电路中各元件吸收的有功功率、无功功率及电源提供的功率。

解:

$$\dot{I}_1 = \frac{100\sqrt{2}\angle 45^\circ}{100 + j300} = \frac{\sqrt{5}}{5} \angle -26.6^\circ \text{ A} = 0.4 - j0.2$$

$$\dot{I}_2 = \frac{100\sqrt{2}\angle 45^\circ}{200 - j400} = \frac{\sqrt{10}}{10} \angle 108.4^\circ \text{ A} = -0.1 + j0.3$$



$$\tilde{S}_1 = \dot{U}_s \dot{I}_1^* = 100\sqrt{2}\angle 45^\circ \cdot \frac{\sqrt{5}}{5} \angle 26.6^\circ = 20 + j60 \text{ VA} = (100 + j100)(0.4 + j0.2)$$

$$\tilde{S}_2 = \dot{U}_s \dot{I}_2^* = 100\sqrt{2}\angle 45^\circ \cdot \frac{\sqrt{10}}{10} \angle -108.4^\circ = 20 - j40 \text{ VA} = (100 + j100)(-0.1 - j0.3)$$

$$P_{100\Omega} = 20 \text{ W}, \quad Q_{100\Omega} = 0 \text{ Var}, \quad P_{200\Omega} = 20 \text{ W}, \quad Q_{200\Omega} = 0 \text{ Var}$$

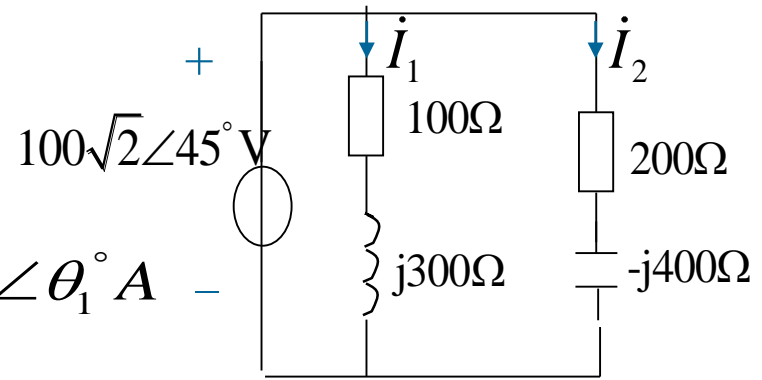
$$P_L = 0 \text{ W}, \quad Q_L = 60 \text{ Var}, \quad P_C = 0 \text{ W}, \quad Q_C = -40 \text{ Var}$$

$$P_{U_s} = P_{100\Omega} + P_{200\Omega} = 40 \text{ W}$$

另解:

$$\dot{I}_1 = \frac{100\sqrt{2}\angle 45^\circ}{100 + j300} = \frac{100\sqrt{2}}{100\sqrt{10}} \angle \theta_1^\circ = \frac{1}{\sqrt{5}} \angle \theta_1^\circ \text{ A}$$

$$\dot{I}_2 = \frac{100\sqrt{2}\angle 45^\circ}{200 - j400} = \frac{100\sqrt{2}}{100\sqrt{20}} \angle \theta_2^\circ = \frac{1}{\sqrt{10}} \angle \theta_2^\circ \text{ A}$$



$$P_{100\Omega} = I_1^2 \times 100 = 20\text{W}, \quad Q_{100\Omega} = 0\text{Var}$$

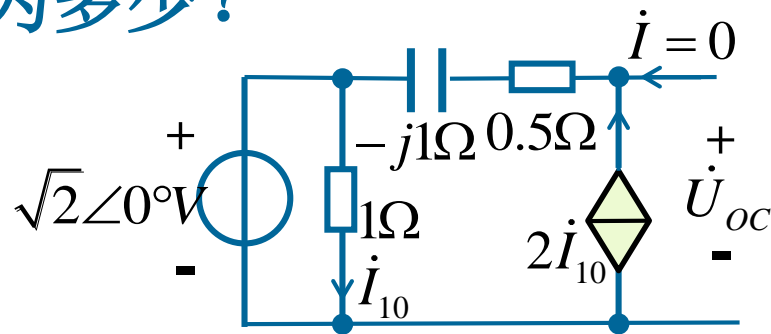
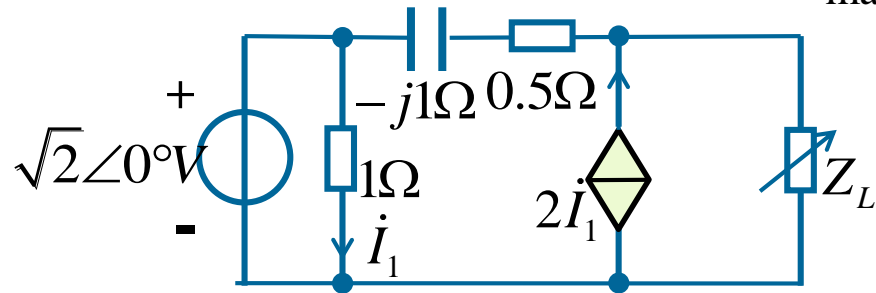
$$P_{200\Omega} = I_2^2 \times 200 = 20\text{W}, \quad Q_{200\Omega} = 0\text{Var}$$

$$P_L = 0\text{W}, \quad Q_L = I_1^2 \times 300 = 60\text{Var}$$

$$P_C = 0\text{W}, \quad Q_C = I_2^2 \times (-400) = -40\text{Var}$$

$$P_{U_s} = P_{100\Omega} + P_{200\Omega} = 40\text{W}$$

7-30 电路如题图7-30所示，试求负载 Z_L 为何值时可获得最大功率？最大功率 P_{\max} 为多少？



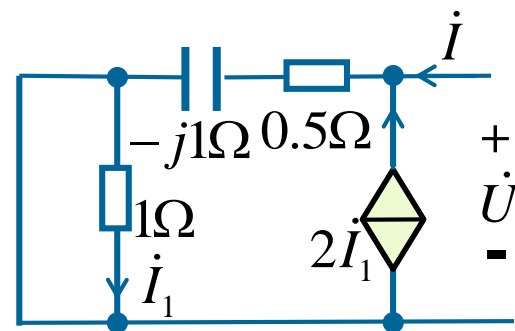
解： $\dot{U}_{oc} = 2\dot{I}_{10}(0.5 - j1) + \sqrt{2} \angle 0^\circ$

$$\dot{I}_{10} = \frac{\sqrt{2} \angle 0^\circ}{1} = \sqrt{2} \angle 0^\circ \text{ A}$$

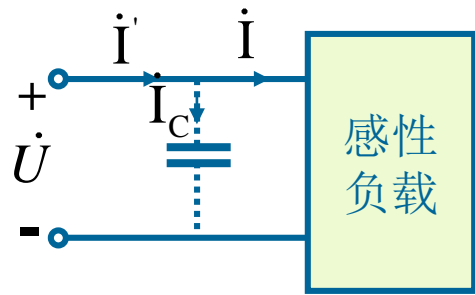
$$\therefore \dot{U}_{oc} = 2\sqrt{2} - j2\sqrt{2} = 4 \angle -45^\circ \text{ V}$$

$$\dot{I}_1 = 0 \text{ A} \Rightarrow 2\dot{I}_1 = 0 \quad \therefore Z_0 = 0.5 - j1\Omega$$

当 $Z_L = Z_0^* = 0.5 + j1\Omega$ 时，可获得最大功率：
$$P_{\max} = \frac{U_{oc}^2}{4R_0} = 8 \text{ W}$$



7-28 如题图7-28所示，已知某感性负载接于电压220V、频率50Hz的交流电源上，其吸收的平均功率为40W，端口电流 $I=0.66A$ ，试求感性负载的功率因数；如欲使电路的功率因数提高到0.9，问至少需并联多大电容 C ？



解： $\because P = UI \cos \theta_Z$

故： $p_f = \cos \theta_Z = \frac{P}{UI} = \frac{40}{220 \times 0.66} \approx 0.275$

且 $\theta_Z = 74^\circ$

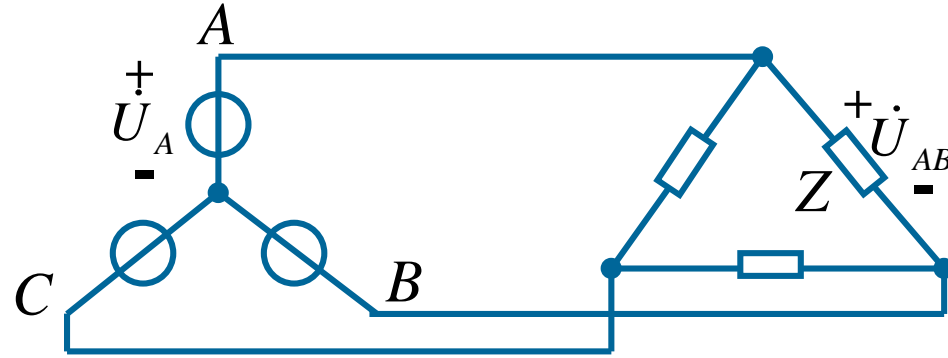
当 $p'_f = \cos \theta'_Z = 0.9$, $I' = \frac{P}{U \cdot p'_f} = \frac{40}{220 \times 0.9} \approx 0.202A$

且 $\theta_Z = 25.8^\circ$ 设 $\dot{U} = 220 \angle 0^\circ$

$\therefore \dot{I}_C = \dot{I}' - \dot{I} = 0.202 \angle -25.8^\circ - 0.66 \angle -74^\circ \approx j0.55A$

$\because I_C = \omega CU \Rightarrow C = \frac{I_C}{2\pi fU} = \frac{0.55}{2 \times 3.14 \times 50 \times 220} \approx 7.9 \mu f$

7-33 已知三角形连接的对称负载接于对称星形连接的三相电源上，若每相电源相电压为220V，各相负载阻抗 $Z = 30 + j40\Omega$ ，试求负载相电流和线电流的有效值。



解：连接电路如图所示，且已知电源相电压，即：

$$U_A = U_P = 220V$$

则电源的线电压为：

$$U_l = U_{AB} = \sqrt{3}U_P = 380V$$

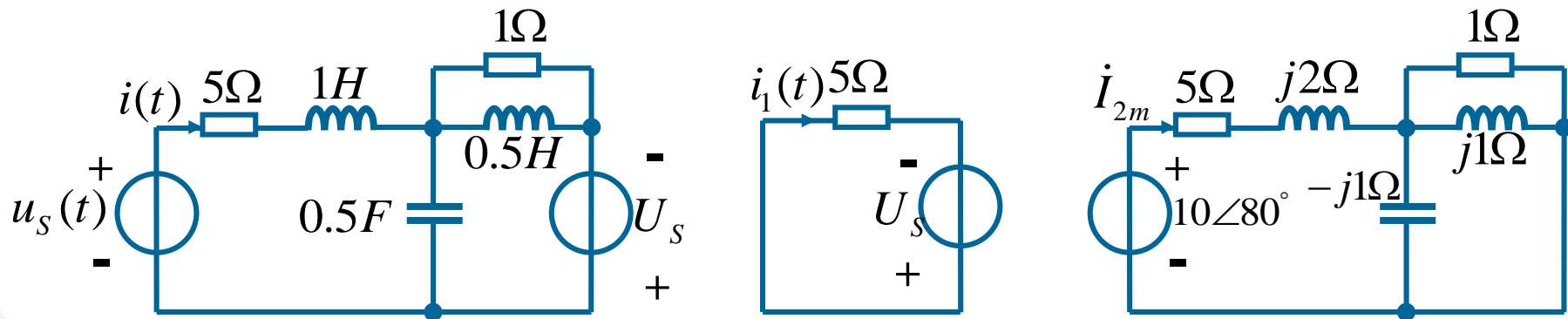
故负载 Z 上的相、线电流为：

$$I_{AB} = \frac{U_{AB}}{|Z|} = \frac{380}{\sqrt{30^2 + 40^2}} = 7.6A = I_p$$

$$\therefore I_l = \sqrt{3}I_p \approx 7.6\sqrt{3} = 13.2A$$

思考：总的吸收功率？

7-35 稳态电路如题图7-35所示，已知 $u_s(t) = 10\cos(2t + 80^\circ)V$ ， $U_s = 10V$ ，试求电流 $i(t)$ 。



解：应用叠加定理，如图所示，则：

$$i_1(t) = \frac{U_s}{5} = 2A$$

$$\dot{I}_{2m} = \frac{10\angle 80^\circ}{5 + j2 + (-j1) // j1 // 1} \approx 1.6\angle 61.6^\circ A$$

$$\therefore i_2(t) = 1.6\cos(2t + 61.6^\circ)A$$

$$\therefore i(t) = i_1(t) + i_2(t) = 2 + 1.6\cos(2t + 61.6^\circ)A$$

7-41 题图7-41所示二端网络N的端口电流、电压分别为

$$i(t) = 5 \cos t + 2 \cos\left(2t + \frac{\pi}{4}\right) \text{A},$$
$$u(t) = 3 + \cos\left(t + \frac{\pi}{2}\right) + \cos\left(2t - \frac{\pi}{4}\right) + \cos\left(3t - \frac{\pi}{3}\right) \text{V}$$

试求网络吸收的平均功率。

解：

$$P = \sum_{k=0}^3 U_k I_k \cos \theta_{zk}$$
$$= U_0 I_0 + U_1 I_1 \cos \frac{\pi}{2} + U_2 I_2 \cos\left(-\frac{\pi}{2}\right)$$
$$= 3 \times 0 + \frac{1}{\sqrt{2}} \cdot \frac{5\sqrt{2}}{2} \cos \frac{\pi}{2} + \frac{1}{\sqrt{2}} \cdot \sqrt{2} \cos\left(-\frac{\pi}{2}\right) = 0$$

7-42 已知流过 2Ω 电阻的电流

$$i(t) = 2 + 2\sqrt{2} \cos t + \sqrt{2} \cos(2t + 30^\circ) A,$$

试求电阻消耗的平均功率。

解:

$$P = \sum_{k=0}^2 I_k^2 R$$

$$= (I_0^2 + I_1^2 + I_2^2) R = (2^2 + 2^2 + 1^2) \times 2 = 18W$$

第8章 耦合电感和变压器电路分析

一、耦合电感的伏安关系

1、自感电压的符号取决于流经电感的电压、电流是否关联，关联取正号，否则取负号；

$$u_1 = \pm L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

$$u_2 = \pm L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt}$$

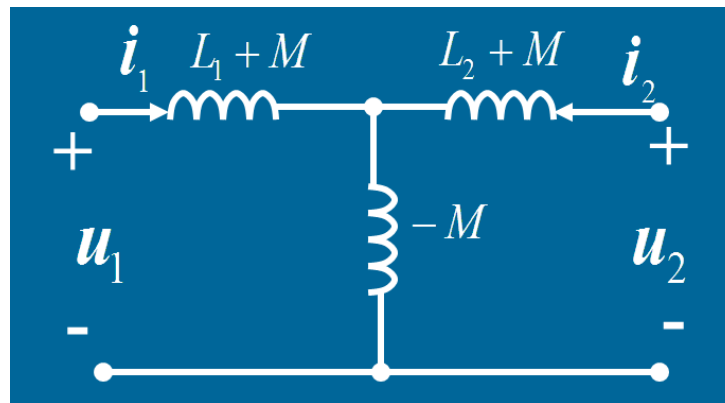
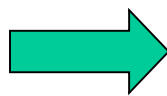
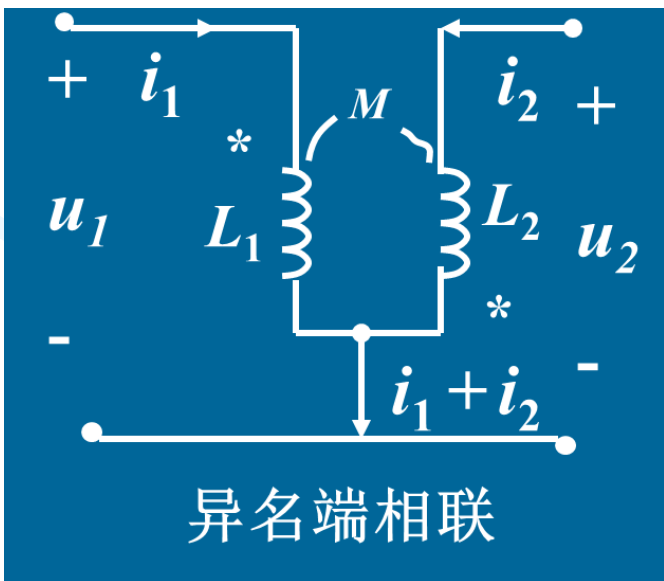
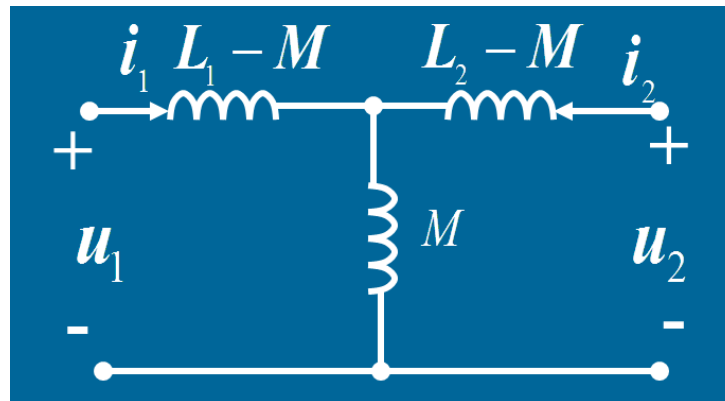
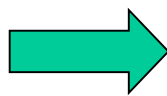
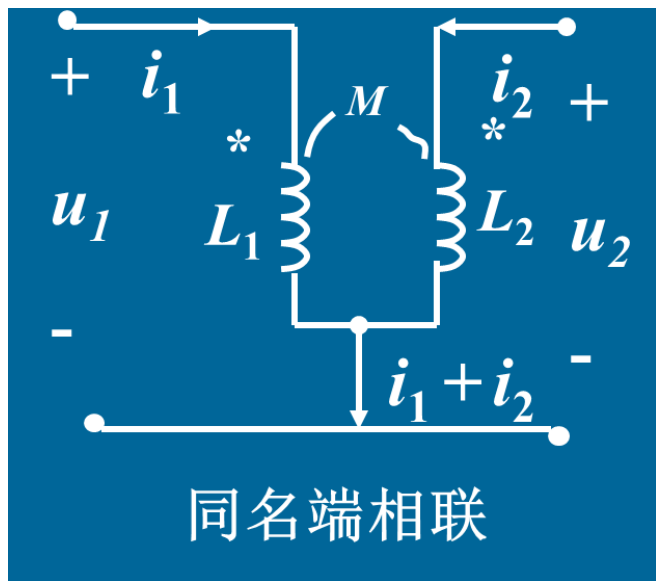
2、当电感电压的正极性端与产生互感电压的另一线圈的电流流入端为同名端时，互感电压取正号，否则取负号。

二、耦合电感的直接去耦等效

串联： $L_{eq} = L_1 + L_2 \pm 2M$ (顺串取+，反串取-)

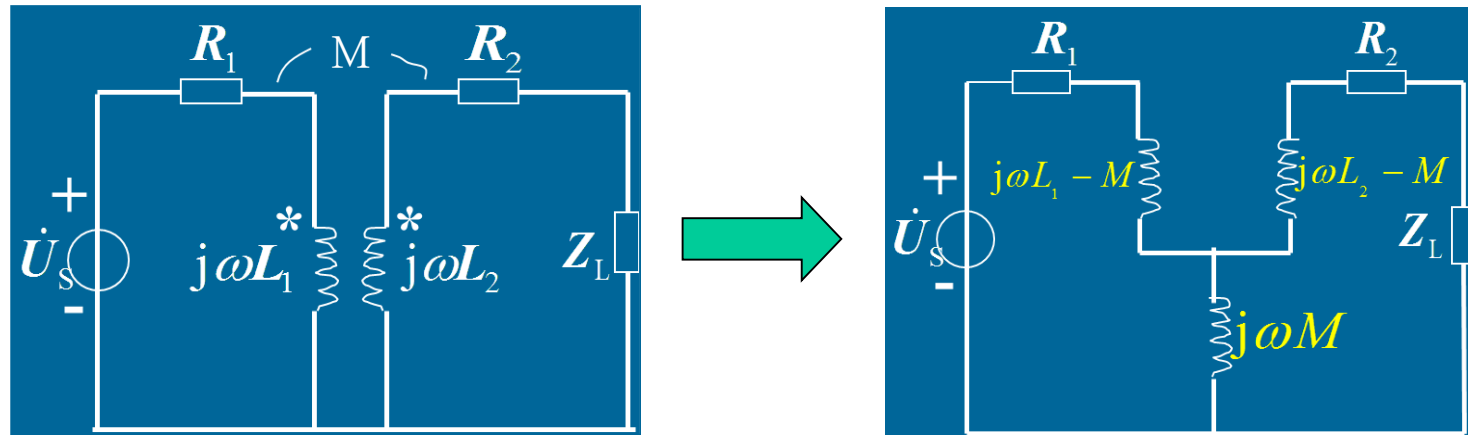
并联： $L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 \mp 2M}$ (顺并取得-，反并取得+)

三端连接：



三、空芯变压器电路的分析

1、直接去耦等效；

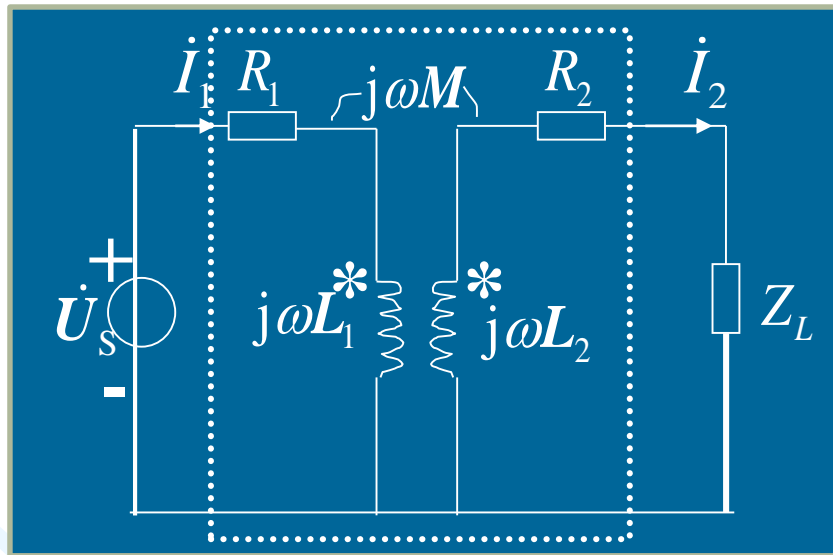


对去耦后的相量模型，应用电路分析的基本依据 (KVL、KCL和元件的VCR)，以及电阻电路中的各种分析法、等效变换和定理，进行相量分析；

将求得的响应的相量表示转换成相应的时域表达式。

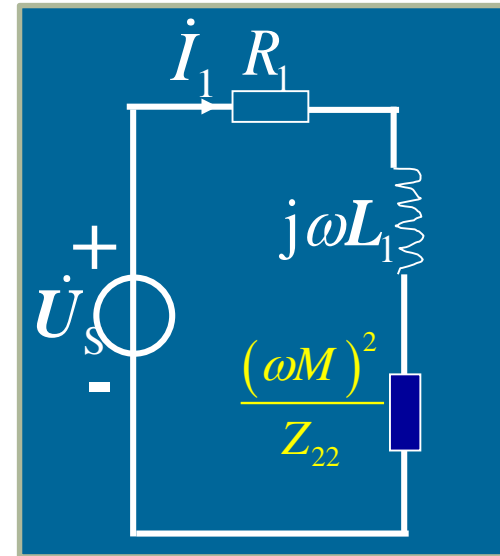
三、空芯变压器电路的分析

2、反映阻抗法；

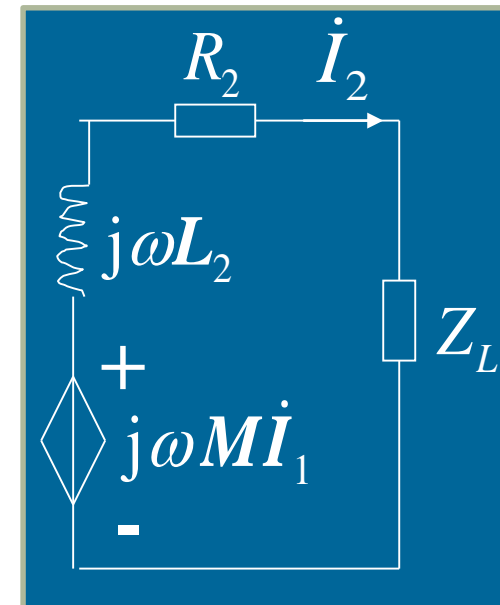


$$\dot{I}_1 = \frac{\dot{U}_s}{Z_{11} + \frac{(\omega M)^2}{Z_{22}}}$$

$$\dot{I}_2 = \frac{j\omega M \dot{I}_1}{Z_{22}}$$



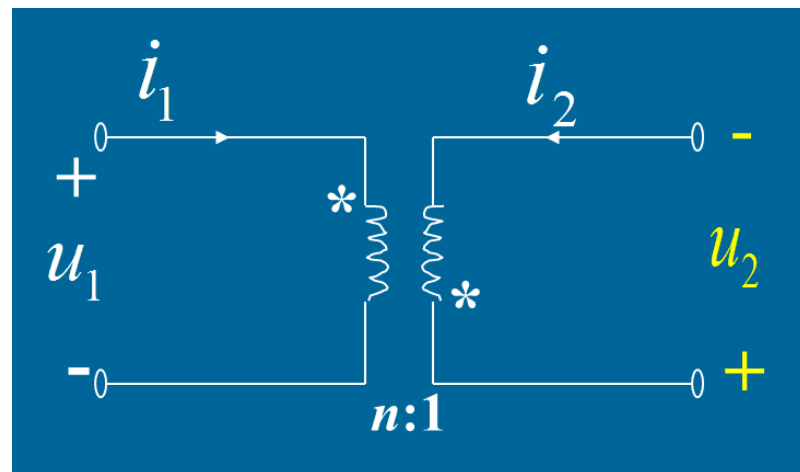
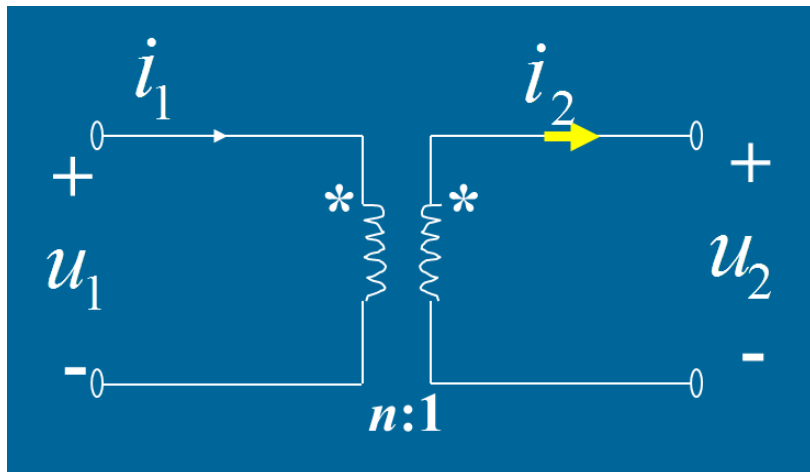
初级等效电路



次级等效电路

四、理想变压器电路

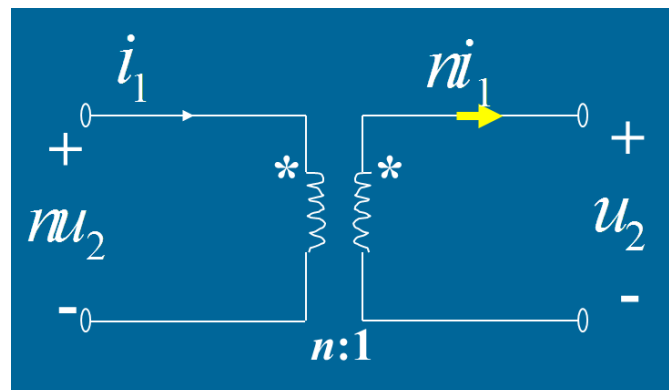
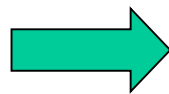
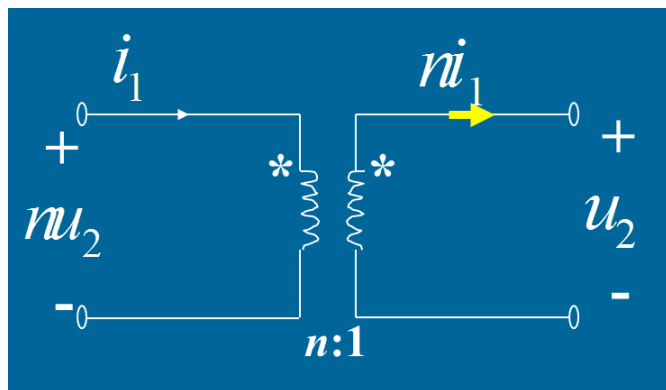
1、理想变压器的伏安关系



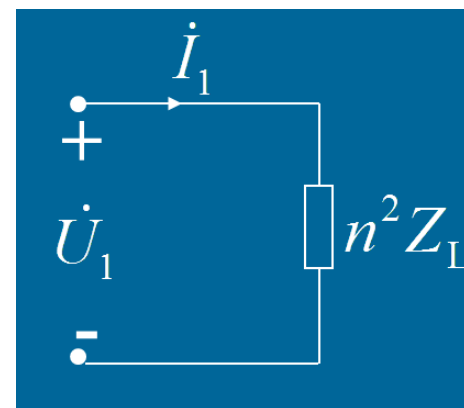
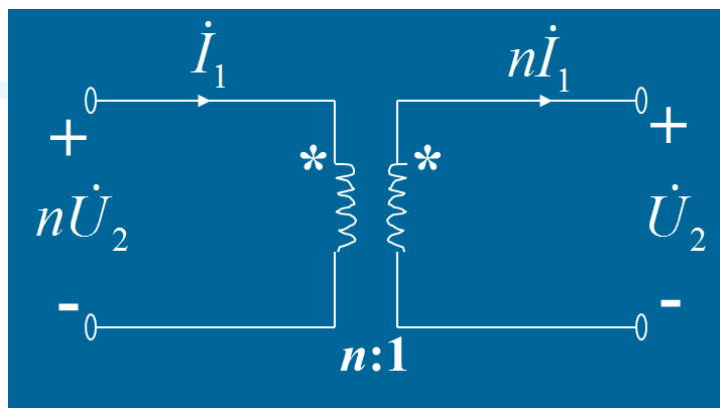
$$\frac{u_1}{u_2} = n, \quad \frac{i_1}{i_2} = \frac{1}{n}$$

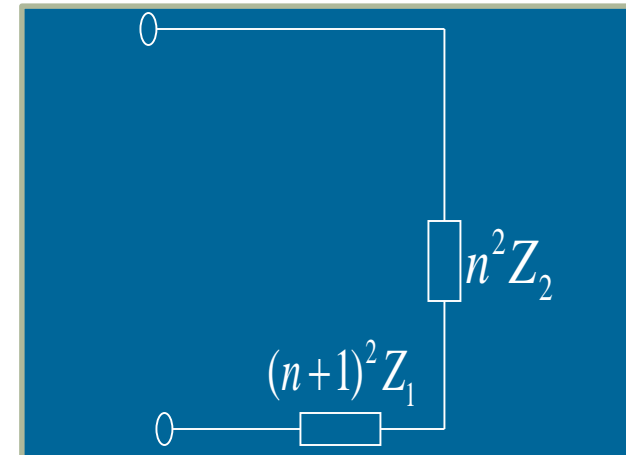
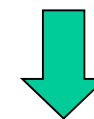
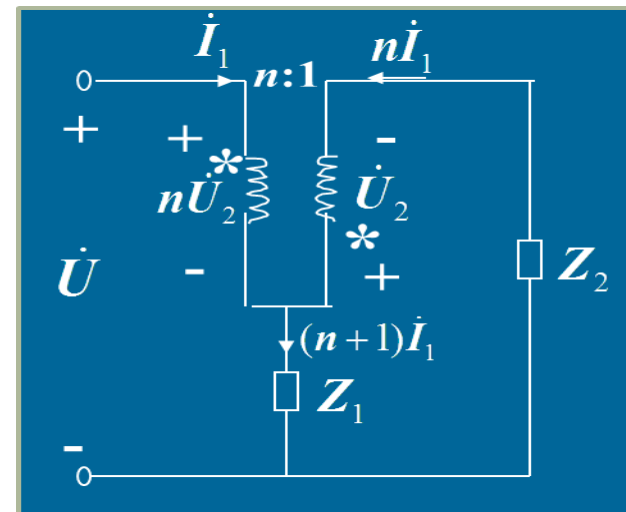
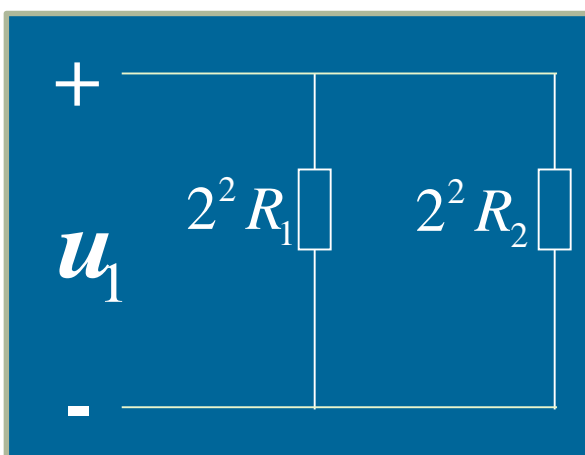
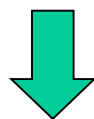
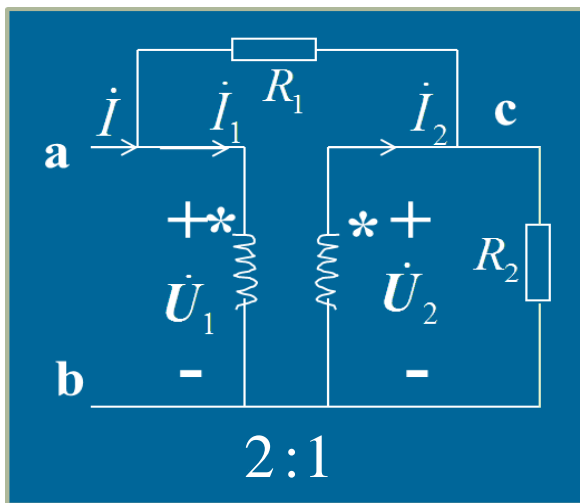
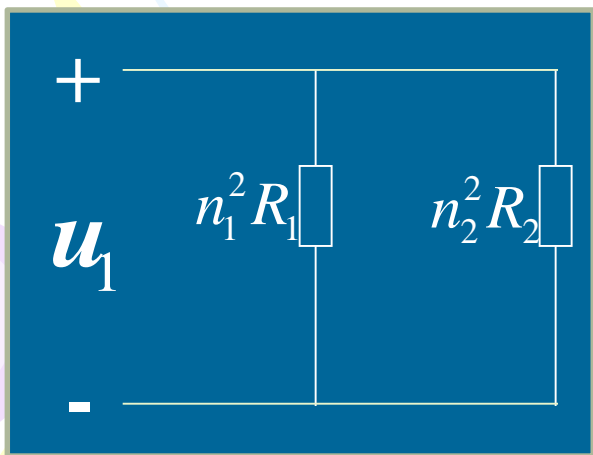
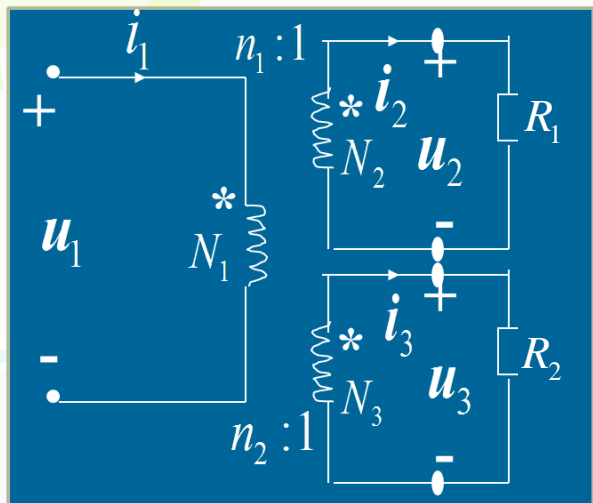
2、理想变压器电路的分析

- **直接用伏安关系**：然后对两个回路分别列KVL或网孔方程求解 i_1 和 u_2 。

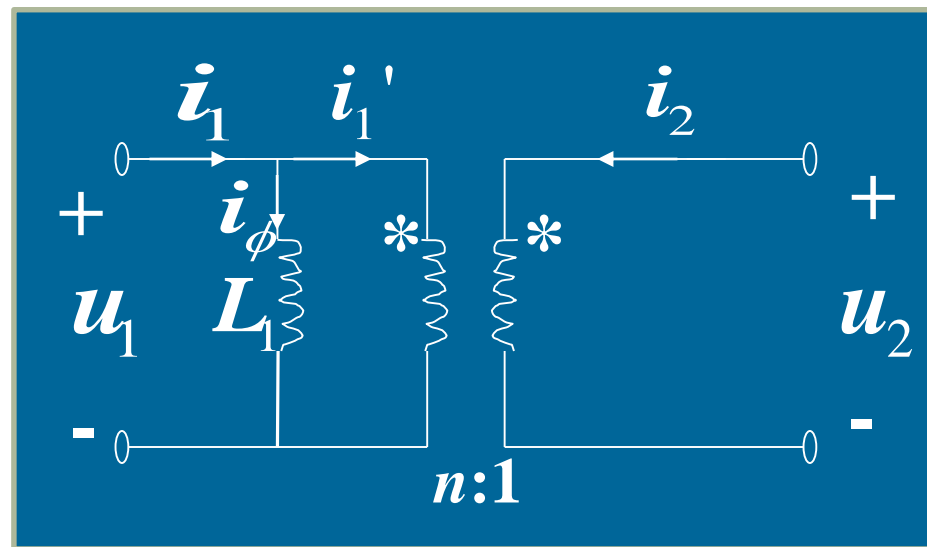
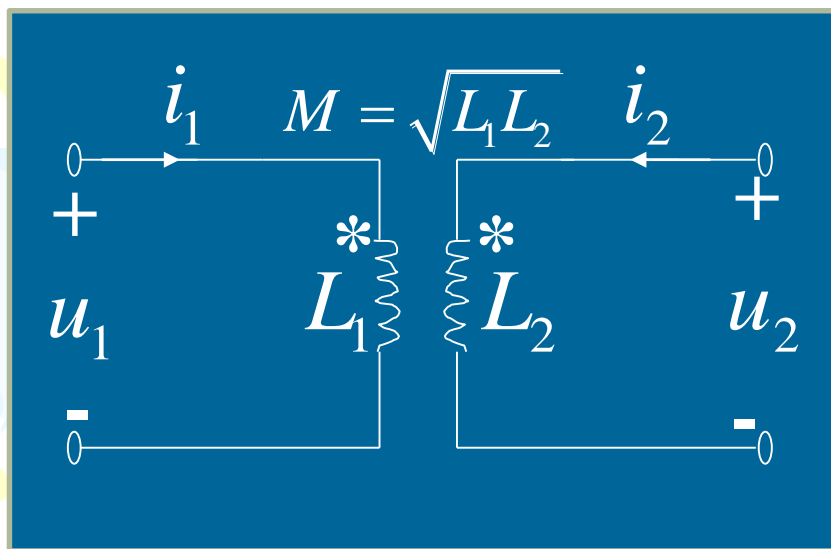


- **阻抗搬移**：按平方倍搬移阻抗，然后对等效电路进行求解。



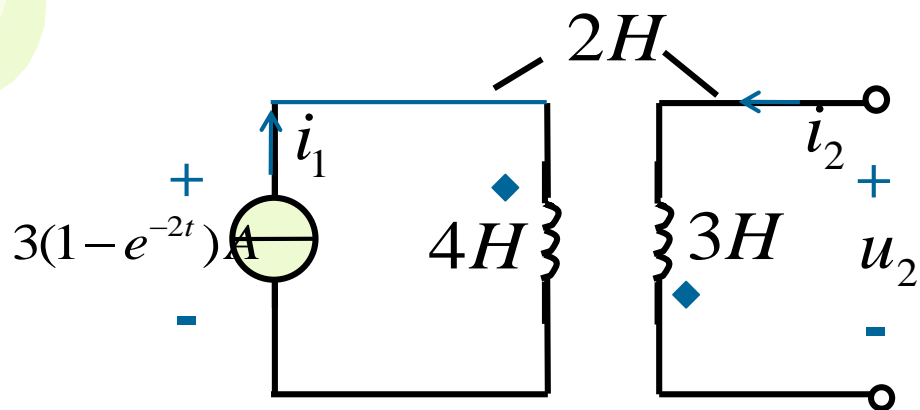


五、全耦合变压器的电路模型

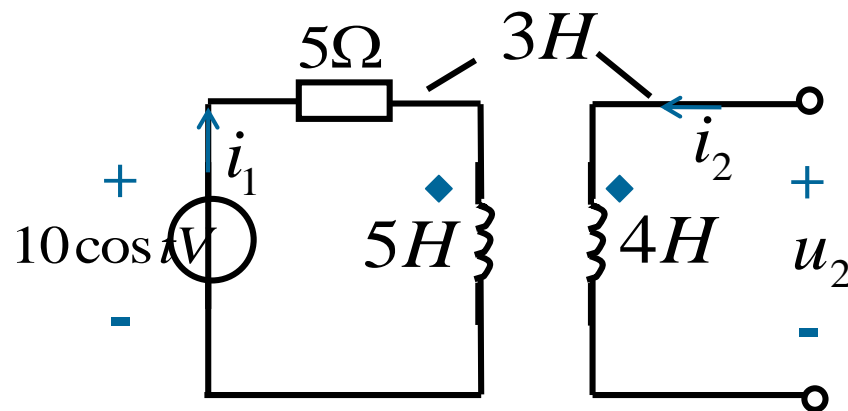


$$n = \sqrt{\frac{L_1}{L_2}}$$

8-3 试求题图8-3中的电压 u_2 。



题图8-3(a)



题图8-3(b)

解: (a) $\because i_2 = 0$, 故:

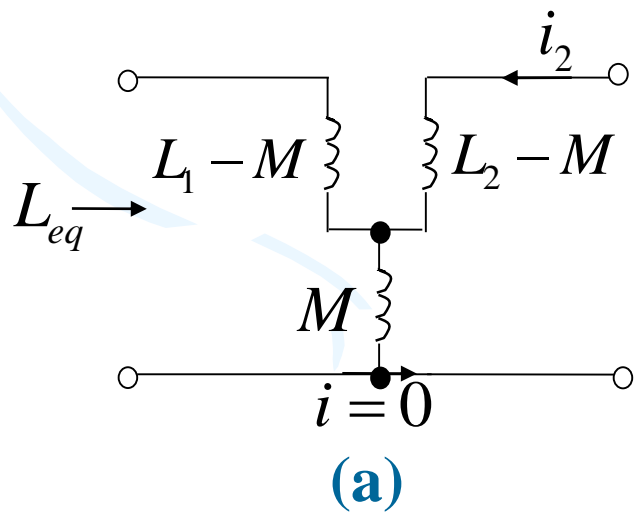
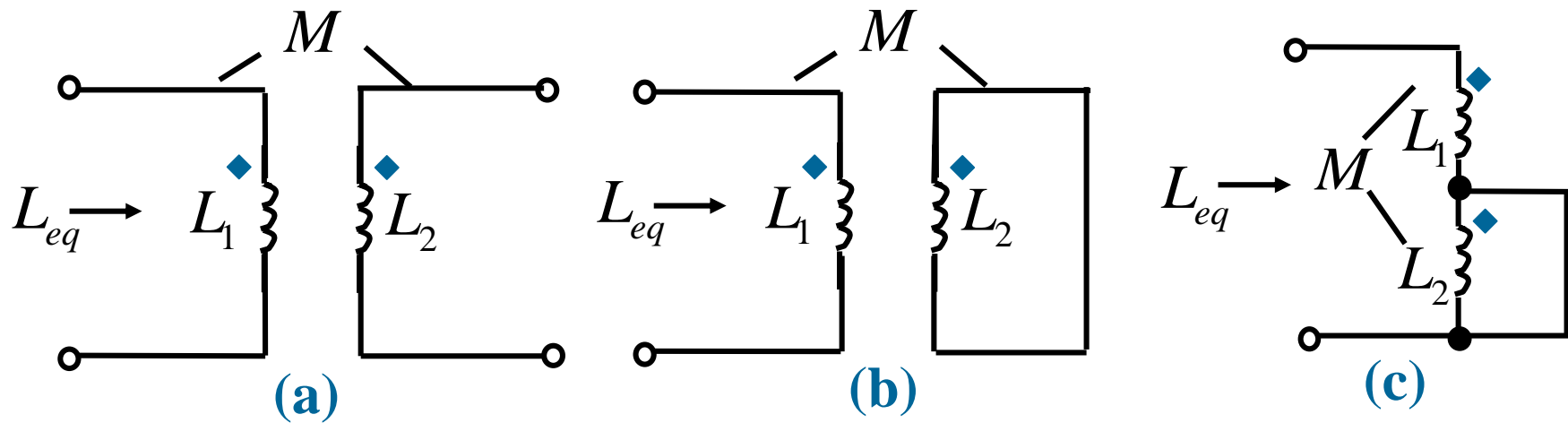
$$u_2 = -M \frac{di_1}{dt} = -2 \frac{d[3(1 - e^{-2t})]}{dt} = -12e^{-2t} \text{V}$$

(b) $\because i_2 = 0$, 故: $\dot{I}_1 = \frac{5\sqrt{2}\angle 0^\circ}{5 + j5} = 1\angle -45^\circ \text{A}$

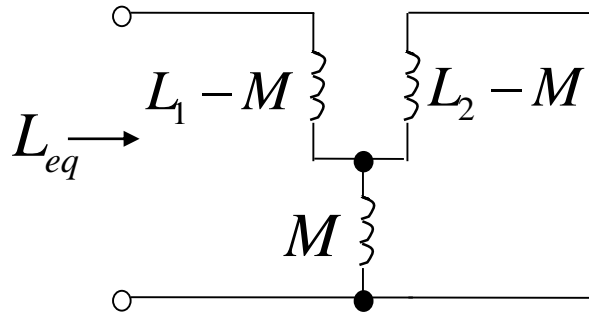
$$\therefore \dot{U}_2 = j\omega M \dot{I}_1 = 3\angle 45^\circ \text{V}$$

$$u_2 = 3\sqrt{2} \cos(t + 45^\circ) \text{V}$$

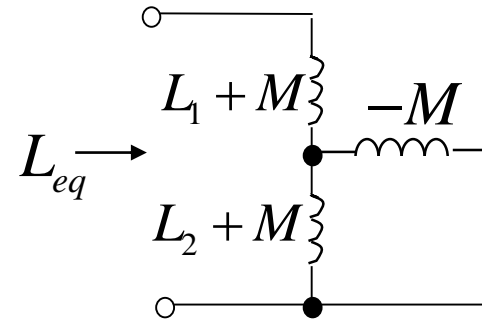
8-4 耦合电感 $L_1 = 6H, L_2 = 4H, M = 2H$ 试求题图8-4中三种连接时的等效电感 L_{eq} 。



$$L_{eq} = L_1 - M + M = L_1 = 6H$$



(b)



(c)

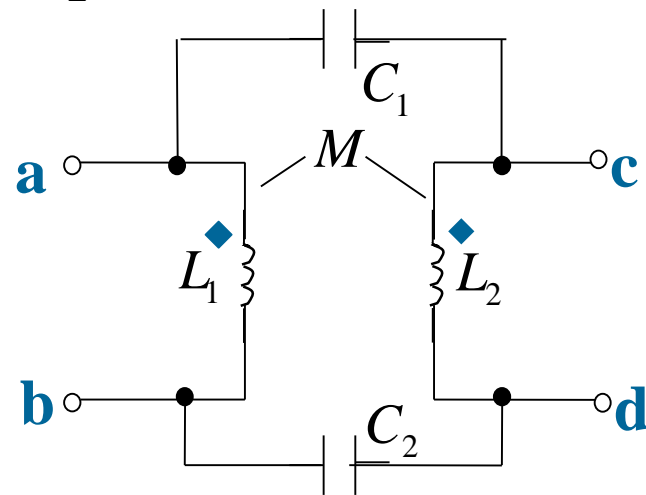
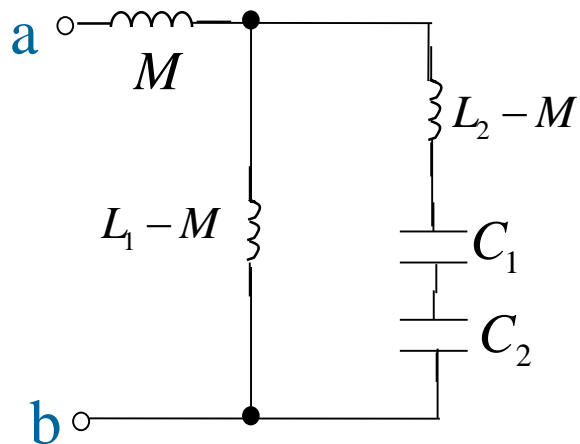
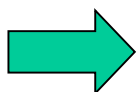
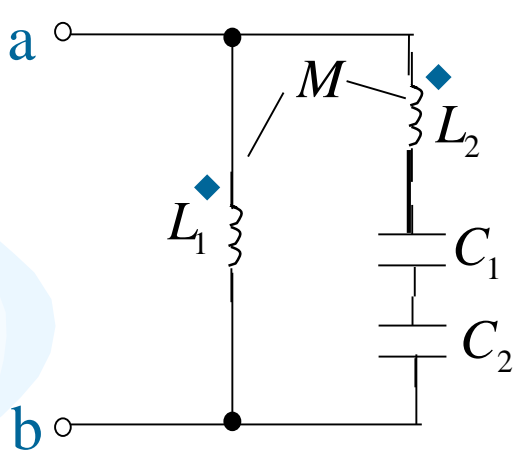
(b)

$$\begin{aligned}
 L_{eq} &= (L_1 - M) + (L_2 - M) // M \\
 &= L_1 - M + \frac{M(L_2 - M)}{L_2 - M + M} = 5H
 \end{aligned}$$

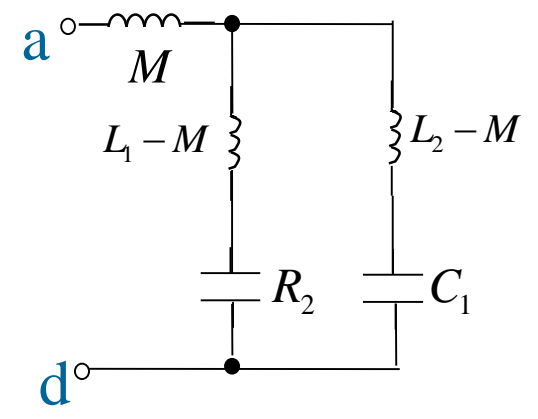
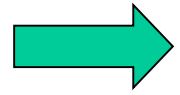
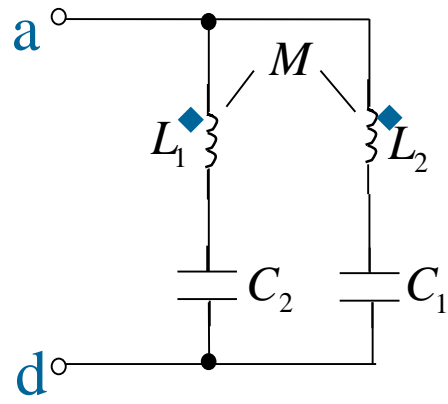
(c)

$$\begin{aligned}
 L_{eq} &= (L_1 + M) + (L_2 + M) // (-M) \\
 &= L_1 + M + \frac{-M(L_2 + M)}{L_2 + M - M} = 5H
 \end{aligned}$$

8-6 电路如题图8-6所示, $\omega = 10^3 \text{ rad/s}$, $L_1 = L_2 = 1\text{H}$,
 $M = 0.5\text{H}$, $C_1 = C_2 = 1\mu\text{F}$, 试求 Z_{ab} 和 Z_{ad} 。

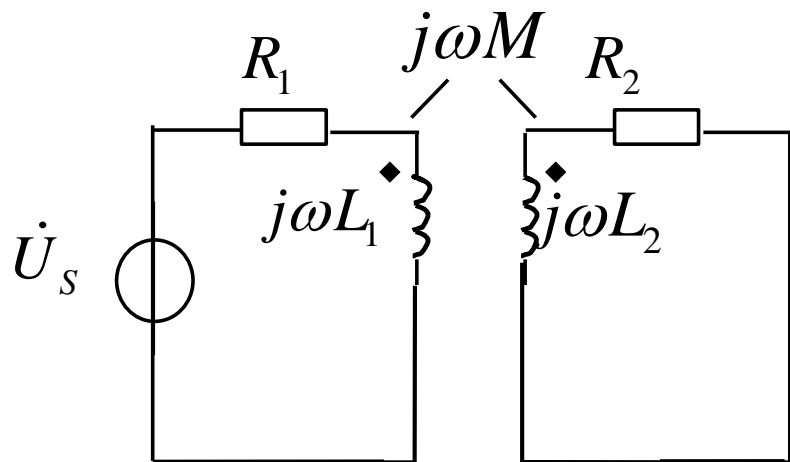


$$\begin{aligned}
 Z_{ab} &= j\omega M + j\omega(L_1 - M) // \left[j\omega(L_2 - M) + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2} \right] \\
 &= j500 + j500 // [j500 - j1000 - j1000] \\
 &= j1250\Omega
 \end{aligned}$$

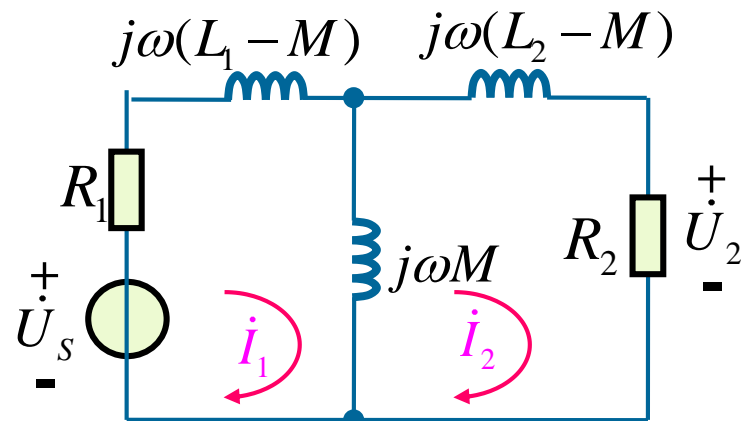


$$\begin{aligned}
 Z_{ad} &= j\omega M + \left[j\omega(L_1 - M) + \frac{1}{j\omega C_2} \right] // \left[j\omega(L_2 - M) + \frac{1}{j\omega C_1} \right] \\
 &= j500 + [j500 - j1000] // [j500 - j1000] \\
 &= j250\Omega
 \end{aligned}$$

8-8 在题图8-8所示电路中, 已知 $R_1 = R_2 = 10\Omega$, $\omega L_1 = 30\Omega$, $\omega L_2 = 20\Omega$, $\omega M = 20\Omega$, $\dot{U}_s = 100\angle 0^\circ\text{V}$ 。试求电压相量 \dot{U}_2 。



题图8-8



解图8-8

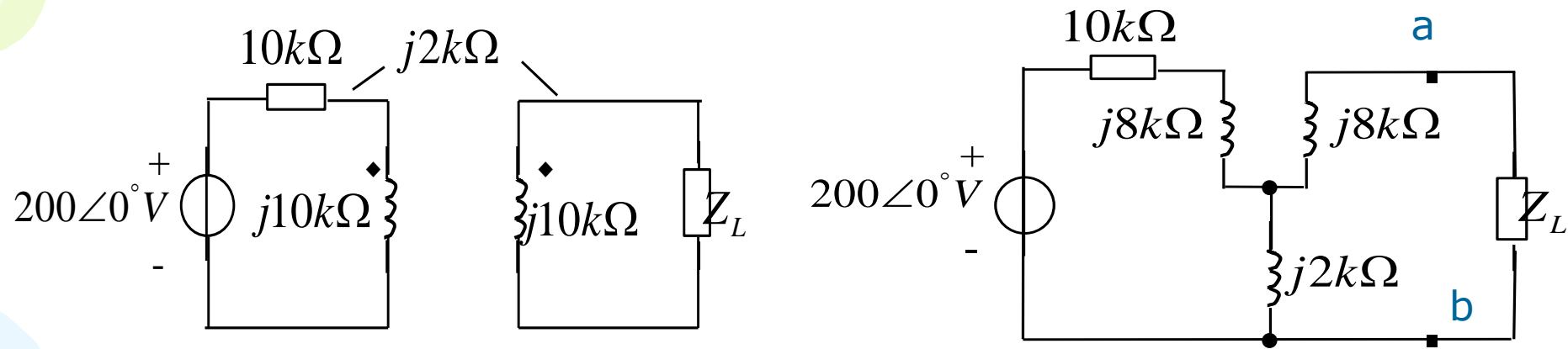
解: L_1 和 L_2 为同名端相连的三端连接, 经去耦等效后如解图8-8:

设网孔电流分别为 i_1 、 i_2 , 则网孔方程为:

$$\begin{cases} [R_1 + j\omega(L_1 - M) + j\omega M] \dot{I}_1 - j\omega M \dot{I}_2 = \dot{U}_s \\ -j\omega M \dot{I}_1 + [R_2 + j\omega(L_2 - M) + j\omega M] \dot{I}_2 = 0 \end{cases}$$

$$\dot{U}_2 = \dot{I}_2 R_2 = 39.2\angle -11.3^\circ$$

8-11 题图8-11所示电路中，试求当 Z_L 为多大时可获得最大功率，以及它获得的最大功率为多少？

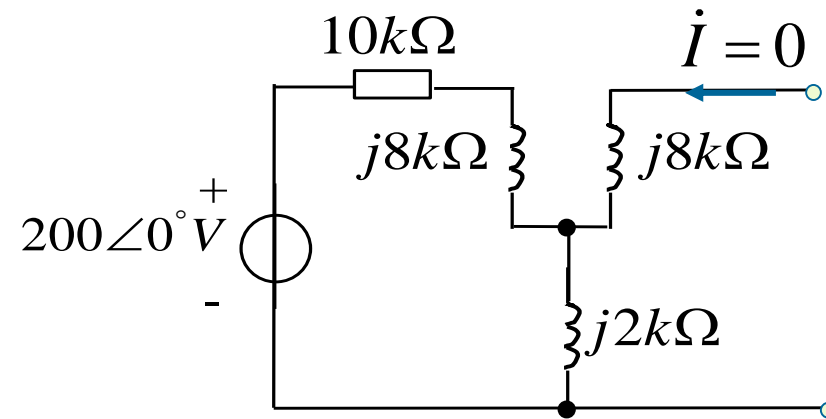


解：去耦等效后的电路如图：

(1) 求 Z_L 以左的等效阻抗 Z_0 ：

$$Z_0 = (10 + j8) // j2 + j8 = (0.2 + j9.8)\text{k}\Omega$$

故， $Z_L = Z_0^* = (0.2 - j9.8)\text{k}\Omega$ 时，可获得最大功率。



解图 8-11

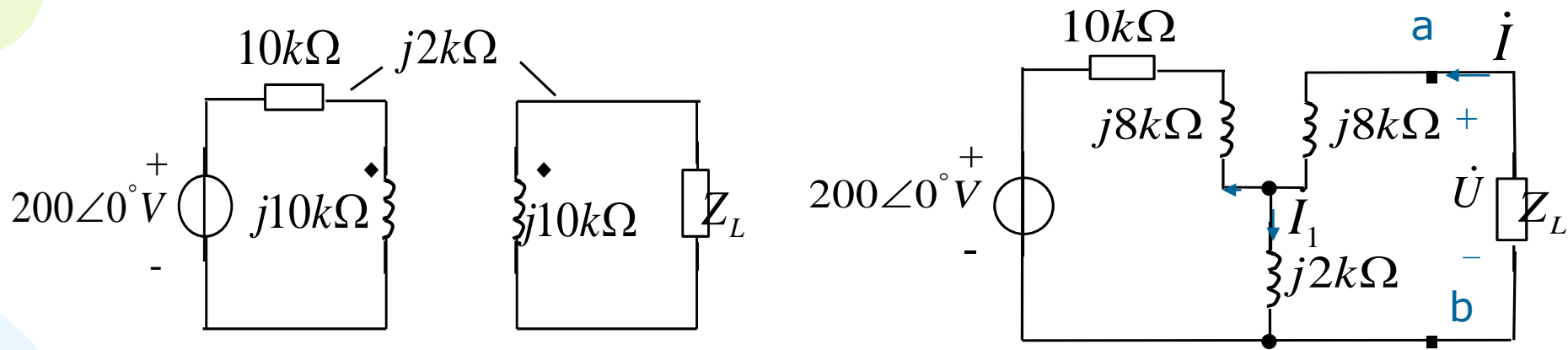
(2) 求 Z_L 以左的开路电压 \dot{U}_{oc} :

$$\dot{U}_{oc} = 200\angle 0^\circ \times \frac{j2}{10 + j8 + j2} = 20\sqrt{2}\angle 45^\circ \text{ V}$$

故, Z_L 可获得的最大功率为:

$$P_{L\max} = \frac{U_{oc}^2}{4R_0} = \frac{(20\sqrt{2})^2}{4 \times 0.2 \times 10^3} = 1 \text{ W}$$

另解：直接对去耦等效后的电路求端口方程：



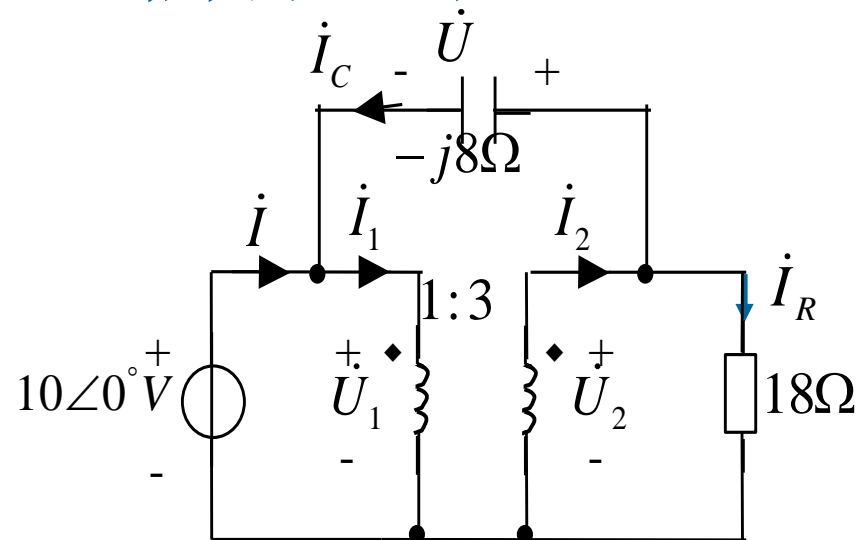
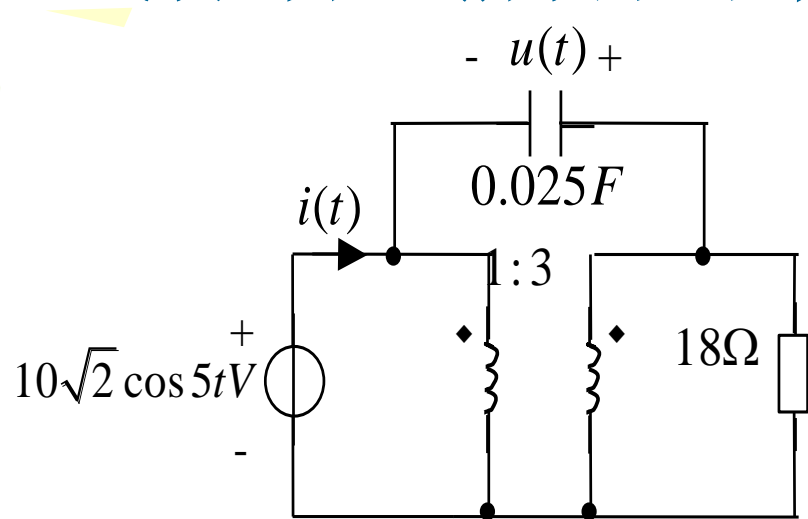
$$\begin{cases} \dot{U} = j8\dot{I} + j2\dot{I}_1 \quad \text{kV} \\ j2\dot{I}_1 = (10 + j8)(\dot{I} - \dot{I}_1) + 0.2 \quad \text{kV} \end{cases}$$

$$\dot{U} = (0.2 + j9.8) \times 10^3 \dot{I} + 20 + j20 \text{V}$$

故， $Z_L = Z_0^* = (0.2 - j9.8) \text{k}\Omega$ 时，可获得最大功率：

$$P_{L\max} = \frac{U_{oc}^2}{4R_0} = \frac{(20\sqrt{2})^2}{4 \times 0.2 \times 10^3} = 1 \text{W}$$

8-13 试求题图8-13所示的正弦稳态电路中的 $i(t)$ 和 $u(t)$ 。



解： 电路的相量模型如解图 8-13 ；

$$\dot{U} = \dot{U}_2 - \dot{U}_1 = 2\dot{U}_1 = 20\angle 0^\circ$$

$$\dot{I}_C = \frac{\dot{U}}{-j8} = \frac{5}{2}\angle 90^\circ \text{ A}, \quad \dot{I}_R = \frac{\dot{U}_2}{18} = \frac{5}{3}\angle 0^\circ \text{ A}$$

$$\therefore \dot{I}_2 = \dot{I}_R + \dot{I}_C = \frac{5}{3} + j\frac{5}{2} \text{ A}$$

$$\dot{I}_1 = 3\dot{I}_2 = 5 + j7.5 \text{ A}$$

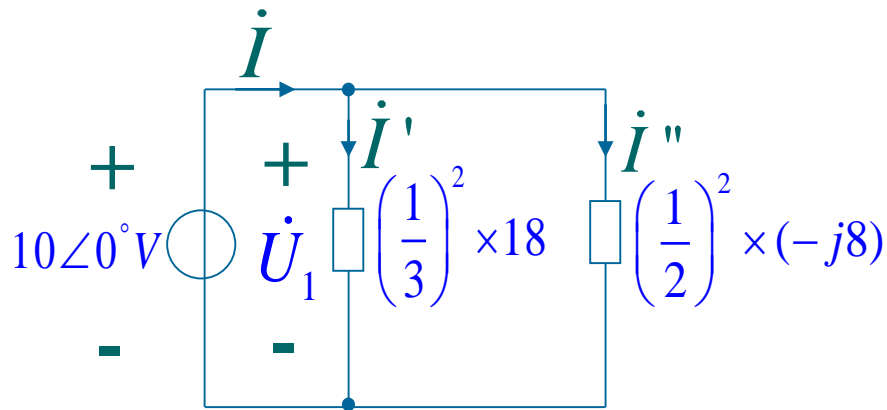
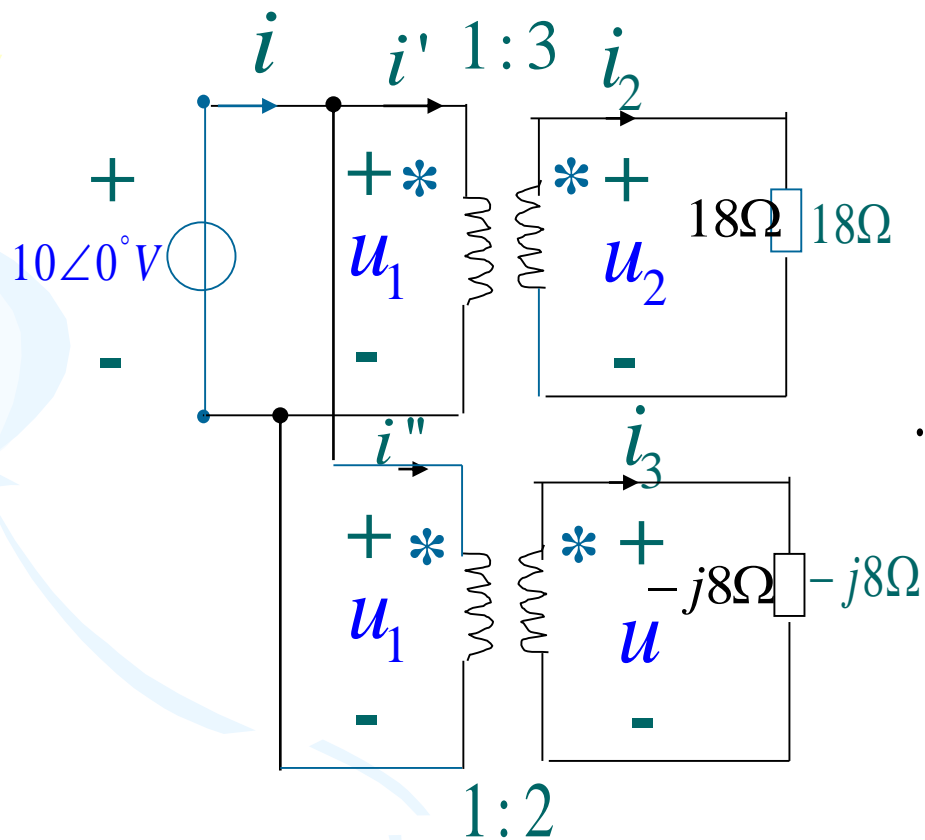
$$\therefore \dot{I} = \dot{I}_1 - \dot{I}_C = 5 + j5 = 5\sqrt{2}\angle 45^\circ \text{ A}$$

$$\therefore u(t) = 20\sqrt{2} \cos 5t \text{ V}$$

$$i(t) = 10 \cos(5t + 45^\circ) \text{ A}$$

另解:

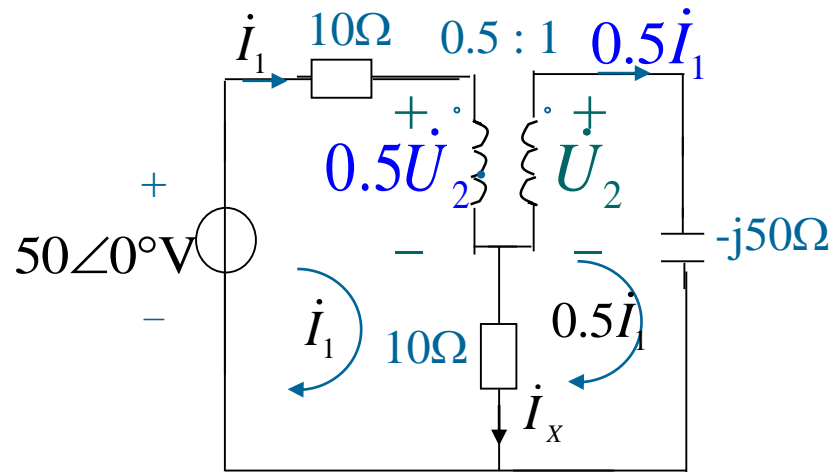
$\because \dot{U} = \dot{U}_2 - \dot{U}_1 = 2\dot{U}_1$ 将初级线圈等效为两线圈并绕。



$$\begin{aligned} \therefore \dot{I} &= \frac{10\angle 0^\circ}{\left(\frac{1}{3}\right)^2 \times 18 // \left(\frac{1}{2}\right)^2 \times (-j8)} \\ &= \frac{10\angle 0^\circ}{1-j1} = 5\sqrt{2}\angle 45^\circ \text{ A} \end{aligned}$$

$$\therefore u(t) = 20\sqrt{2} \cos 5t \text{ V}, i(t) = 10 \cos(5t + 45^\circ) \text{ A}$$

8-14 试求题图 8-14 所示电路中的电流向量 \dot{I}_x 。



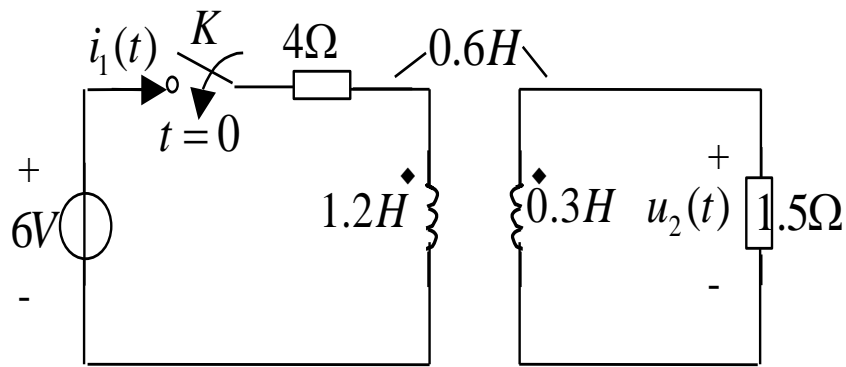
解：理想变压器的问题，一种方法是利用阻抗搬移，另一种方法是可直接利用初、次级线圈间电压、电流的线性关系。

$$\begin{cases} (10+10)\dot{I}_1 - 10 \times 0.5\dot{I}_1 = 50\angle 0^\circ - 0.5\dot{U}_2 \\ -10\dot{I}_1 + (10 - j50) \times 0.5\dot{I}_1 = \dot{U}_2 \end{cases}$$

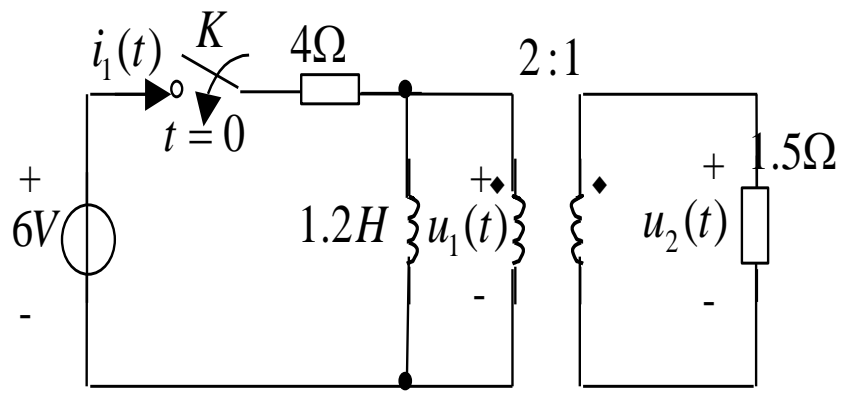
$$\therefore \dot{I}_1 = 2\sqrt{2}\angle 45^\circ \text{ A}$$

$$\dot{I}_x = \dot{I}_1 - 0.5\dot{I}_1 = \sqrt{2}\angle 45^\circ \text{ A}$$

8-20题图8-20所示的电路原已稳定， $t=0$ 时开关K闭合，求 $t>0$ 时电流 $i_1(t)$ 和电压 $u_2(t)$ 。



题图 8-20



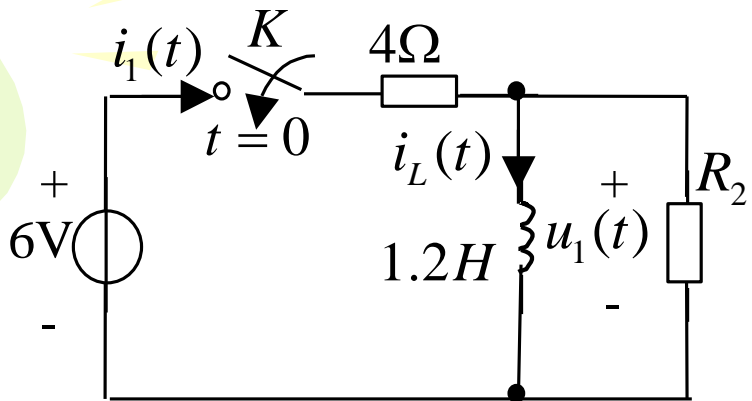
(a)

解图 8-20(1)

解：由于 $M = \sqrt{L_1 L_2} = 0.6H$ ，故为全耦合变压器，电路等效为如解图8-20(1)-(a)；其中：

$$n = \sqrt{\frac{L_1}{L_2}} = 2:1$$

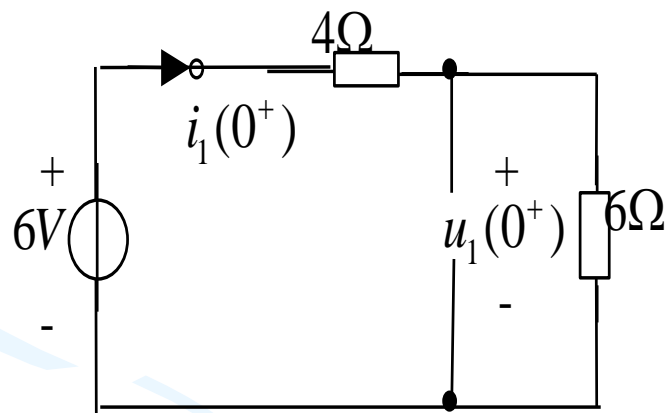
次级线圈的阻抗搬移到初级线圈后的电路模型如解图8-20(1)-(b)；



(b)

其中：搬移后的电阻为 $R_2 = n^2 \cdot 1.5 = 6\Omega$ ；
利用三要素法求解：

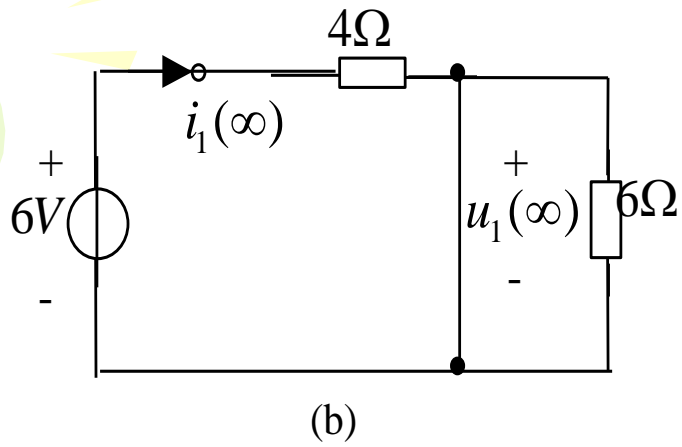
(1) $t = 0^-$ 时, $i_L(0^-) = 0A$ ； $t = 0^+$ 时 $i_L(0^+) = i_L(0^-) = 0A$ ， 则：



(a)

$$i_1(0^+) = \frac{6}{4+6} = 0.6A$$

$$u_1(0^+) = 6 \times i_1(0^+) = 3.6V$$



(2) $t \rightarrow \infty$ 时的电路图如解图8-20(2)-(b), 则可得:

$$i_1(\infty) = \frac{6}{4} = 1.5 \text{ A}$$

$$u_1(\infty) = 0 \text{ V}$$

(3) 求时间常数:

$$R_{eq} = 4 // 6 = 2.4 \Omega$$

$$\tau = \frac{L_1}{R_{eq}} = \frac{1.2}{2.4} = 0.5 \text{ s}$$

(4) 全响应为: $i_1(t) = i_1(\infty) + [i_1(0^+) - i_1(\infty)]e^{-\frac{t}{\tau}} = 1.5 - 0.9e^{-2t} \text{ A}, t > 0$

$$u_1(t) = u_1(\infty) + [u_1(0^+) - u_1(\infty)]e^{-\frac{t}{\tau}} = 3.6e^{-2t} \text{ V}, t > 0$$

$$\therefore u_2(t) = \frac{1}{n} u_1(t) = 1.8e^{-2t} \text{ V}, t > 0$$

第9章 电路的频率特性

一、电路的频率特性与网络函数

电路分析中，电路的频率特性用正弦稳态电路的**网络函数**来描述，定义为：

$$H(j\omega) = \frac{\text{输出相量}}{\text{输入相量}} = |H(j\omega)| \angle \theta(\omega)$$

$|H(j\omega)|$ —幅频特性； $\theta(j\omega)$ —相频特性

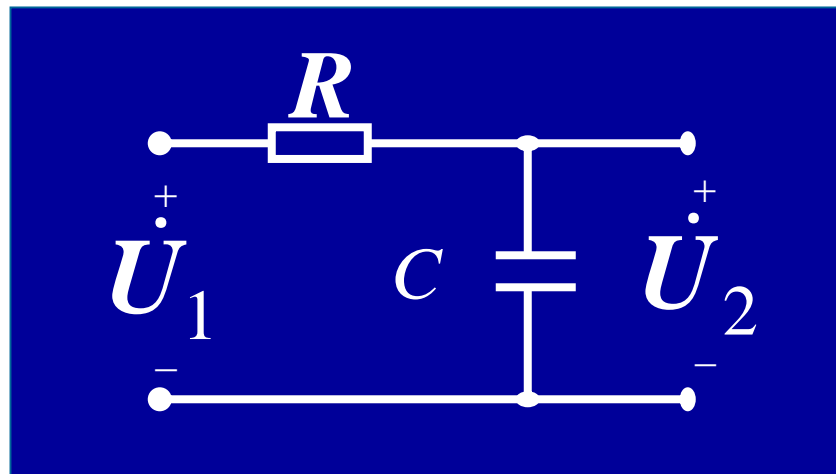
二、RC电路的频率特性

RC低通滤波网络

转移电压比 $|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$

截止角频率 $\omega_c = \frac{1}{RC} \text{ rad/s}$

通频带 $BW = 0 \sim \omega_c$

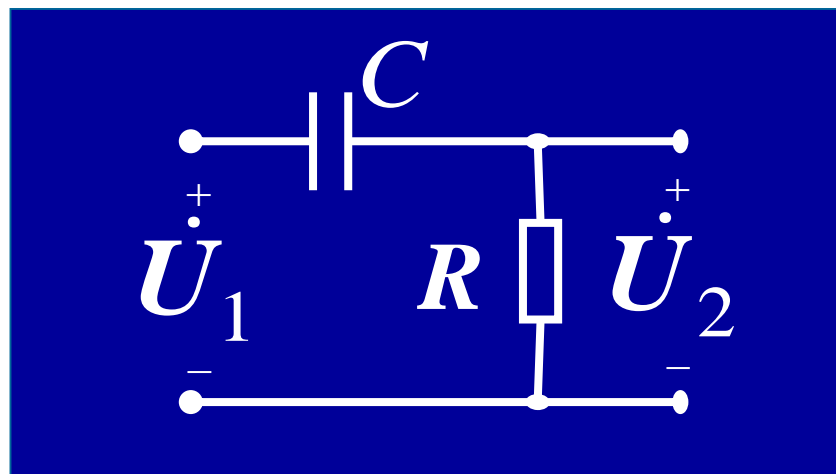


RC高通滤波网络

转移电压比 $|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega_c}{\omega}\right)^2}}$

截止角频率 $\omega_c = \frac{1}{RC} \text{ rad/s}$

通频带 $BW = \omega_c \sim \infty$



三、RLC电路的谐振特性 - RLC串联谐振

谐振角频率 $\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$

品质因数 $Q = \frac{\omega_0 L}{R} = \frac{1}{R\omega_0 C} = \frac{1}{R} \sqrt{\frac{L}{C}}$

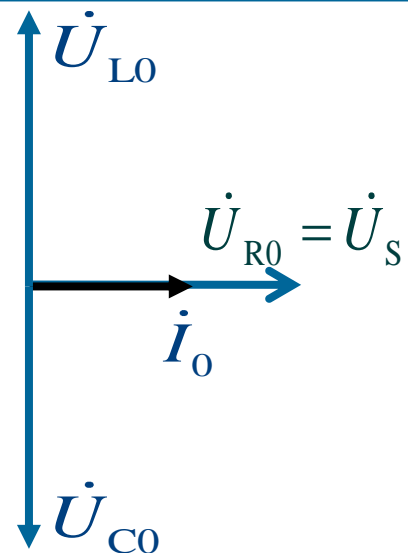
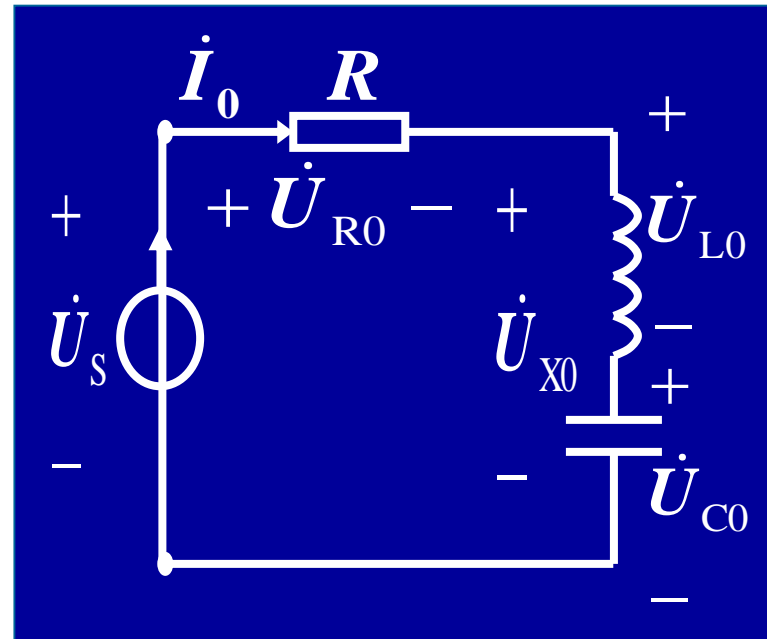
带宽 $BW = \frac{\omega_0}{Q} = \frac{R}{L} \text{ rad/s}$
 $BW = \frac{f_0}{Q} = \frac{R}{2\pi L} \text{ Hz}$

谐振电流 $i_0 = \frac{\dot{U}_S}{R}$

谐振电压 $\dot{U}_{R0} = \dot{U}_S, \dot{U}_{L0} = jQ\dot{U}_S, \dot{U}_{C0} = -jQ\dot{U}_S$

$U_{R0} = U_S, U_{L0} = U_{C0} = QU_S$

电路等效：串联谐振时LC部分相当于**短路**；



三、RLC电路的谐振特性 - GCL并联谐振

谐振角频率 $\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad / s}$

品质因数 $Q = \frac{\omega_0 C}{G} = \frac{1}{\omega_0 GL} = R\sqrt{\frac{C}{L}}$

带宽 $BW = \frac{\omega_0}{Q} = \frac{G}{C} \text{ rad / s}$

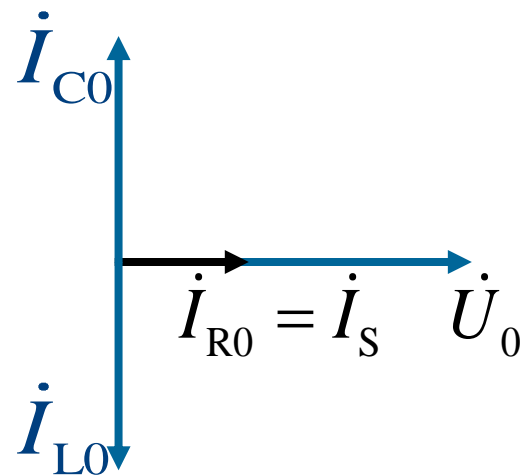
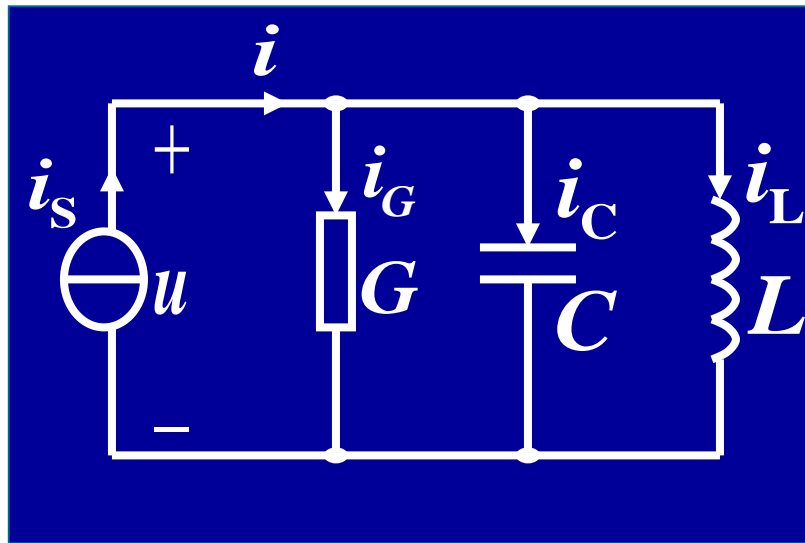
$$BW = \frac{f_0}{Q} = \frac{G}{2\pi C} \text{ Hz}$$

谐振电压 $\dot{U}_0 = \dot{I}_S R$

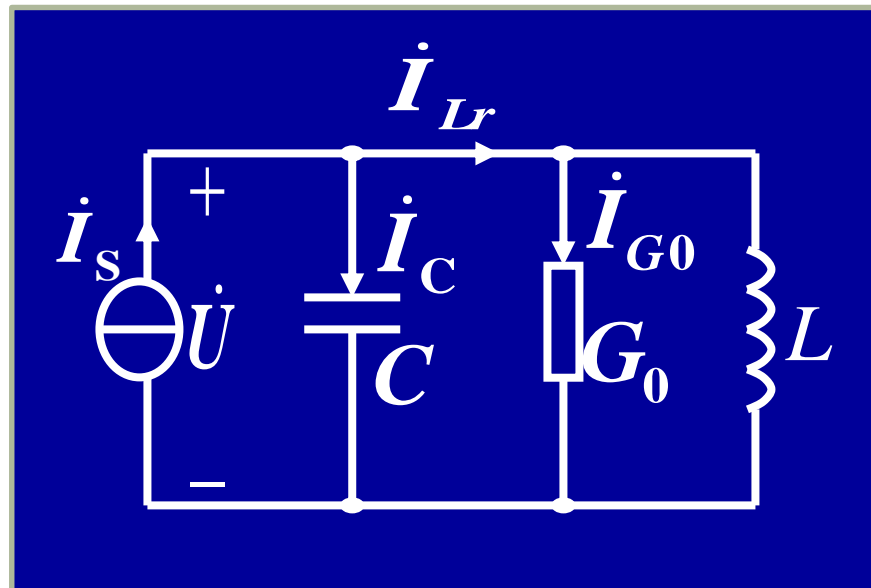
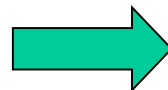
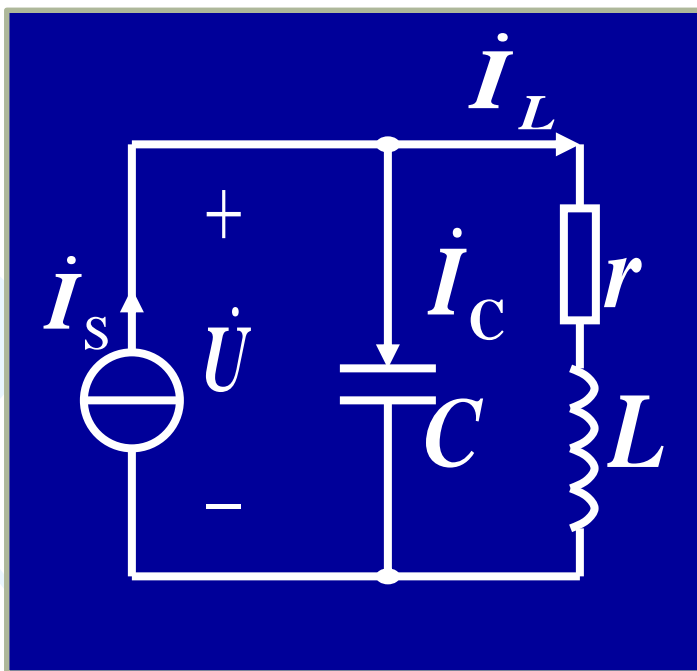
谐振电流 $\dot{I}_{R0} = \dot{I}_S, \dot{I}_{L0} = -jQ\dot{I}_S, \dot{I}_{C0} = jQ\dot{I}_S$

$$I_{R0} = I_S, I_{L0} = I_{C0} = QI_S,$$

电路等效：并联谐振时LC部分相当于开路；



三、RLC电路的谐振特性 - GCL实际并联谐振



$$\omega'_0 \approx \omega_0 = \frac{1}{\sqrt{LC}}$$

$$G_0 = \frac{C}{L} r, \quad Q = \frac{1}{\omega_0 L G_0}$$

(讲义)例4 欲接收载波频率为10MHz的短波电台信号，试设计接收机输入端RLC串联谐振电路的电感线圈。要求带宽 $\Delta f=100\text{kHz}$, $C=100\text{pF}$ 。

解:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$L = \frac{1}{4\pi^2 f_0^2 C} = \frac{1}{4\pi^2 \times 10^{14} \times 10^{-10}} \text{H} = 2.53\mu\text{H}$$

$$\Delta f = \frac{f_0}{Q} \Rightarrow Q = \frac{f_0}{\Delta f} = \frac{10 \times 10^6}{100 \times 10^3} = 100$$

$$R = \frac{1}{Q\omega_0 C} = \frac{1}{100 \times 2\pi \times 10^7 \times 10^{-10}} = 1.59\Omega$$

书例9-1 RLC串联电路, $u_s(t) = \sin(2\pi ft)\text{mV}$, 频率 $f = 1\text{MHz}$, 调电容 C , 使电路发生谐振。 $I_0 = 100\mu\text{A}$, $U_{C0} = 100\text{mV}$ 。求: 电路的 R 、 L 、 C 、 Q 及 BW

解: $U_s = \frac{1}{\sqrt{2}} = 0.707\text{mV}$ $Q = \frac{U_{C0}}{U_s} = \frac{100}{0.707} = 141$

$$R = \frac{U_s}{I_0} = \frac{0.707 \times 10^{-3}}{100 \times 10^{-6}} = 7.07\Omega$$

$$BW = \frac{f_0}{Q} = \frac{10^6}{141} = 7.09\text{kHz}$$

$$BW = \frac{R}{2\pi L} \text{ Hz} \Rightarrow L = \frac{1}{2\pi} \frac{R}{BW} = \frac{7.07}{6.28 \times 7.09 \times 10^3} = 159\mu\text{H}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow C = \frac{1}{\omega_0^2 L} = \frac{1}{(6.28 \times 10^6)^2 \times 159 \times 10^{-6}} = 159\text{pF}$$

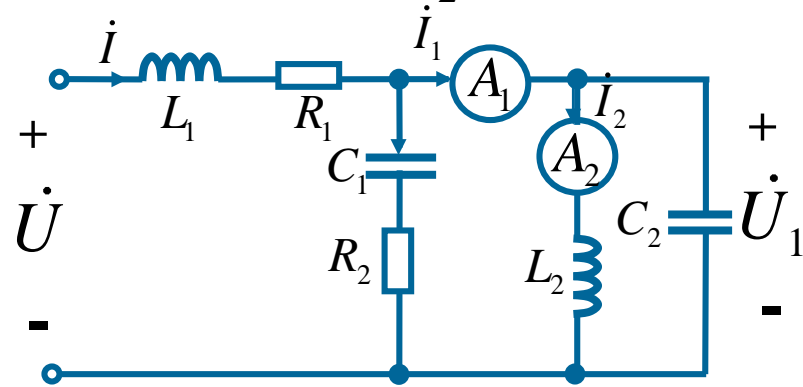
(讲义)例6 GCL并联谐振电路中，已知 $R = 10k\Omega$ **，** $L = 1H$ **，** $C = 1\mu F$ **，试求电路的谐振角频率、品质因数和3dB带宽。**

解：
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 10^{-6}}} \text{ rad/s} = 10^3 \text{ rad/s}$$

$$Q = \frac{\omega_0 C}{G} = R\omega_0 C = R\sqrt{\frac{C}{L}} = 10$$

$$BW = \frac{\omega_0}{Q} = 100 \text{ rad/s}$$

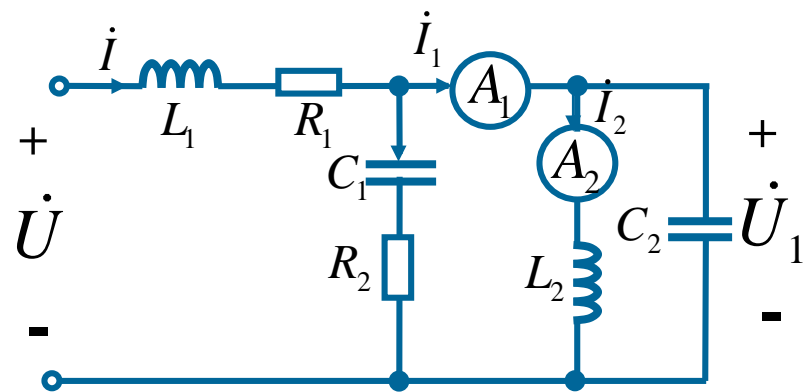
9-11 题图9-11所示电路中，已知 $U = 220V^\circ$, $R_1 = R_2 = 50\Omega$, $L_1 = 0.2H$, $L_2 = 0.1H$, $C_1 = 5\mu F$, $C_2 = 10\mu F$ ，理想电流表 A_1 读数为零，试求理想电流表 A_2 的读数。



解：因为表 A_1 的读数为 **0A**，即 L_2 和 C_2 发生并联谐振，则：

$$\omega_0 = \frac{1}{\sqrt{L_2 C_2}} = 10^3 \text{ rad/s}$$

$\therefore \omega_0 L_1 = \frac{1}{\omega_0 C_1} = 200\Omega$ 故 L_1 和 C_1 发生串联谐振。



设 $\dot{U} = 220 \angle 0^\circ$ ，则：

$$\dot{U}_1 = \dot{U} \times \frac{R_2 + \frac{1}{j\omega_0 C_1}}{R_1 + R_2} = 450 \angle -76^\circ$$

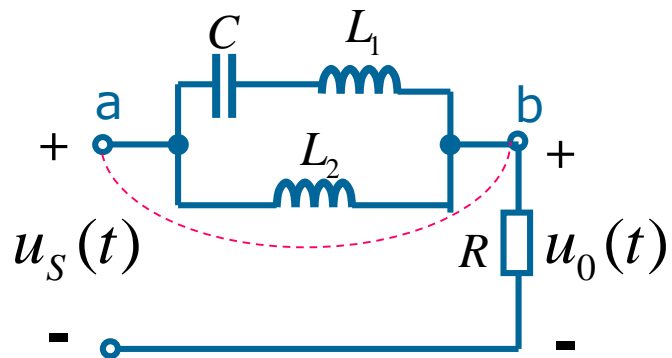
$$\therefore \dot{I}_2 = \frac{\dot{U}_1}{j\omega_0 L_2} = 4.5 \angle -166^\circ \text{ A}$$

故表 A_2 的读数为 **4.5A**

9-14 题图9-14所示电路中，已知

$$u_s(t) = 10 \cos 314t + 2 \cos 3 \times 314t \text{ V}, u_0(t) = 2 \cos 3 \times 314t \text{ V}, C = 9.4 \mu\text{F}$$

试求 L_1 和 L_2 的值。



解： $\omega_1 = 314$ 时， $u_s(t)$ 的该频率分量在 \mathbf{R} 上的电压为零，故，相当于 \mathbf{ab} 端在该频率时开路，即 \mathbf{LC} 回路发生并联谐振，则：

$$\omega_1 = 314 = \frac{1}{\sqrt{(L_1 + L_2)C}} \Rightarrow L_2 = 0.96 \text{ H}$$

$\omega_2 = 3 \times 314$ 时， $u_s(t)$ 的该频率分量全部作用在 \mathbf{R} 上，故，此时 \mathbf{LC} 回路发生串联谐振，即相当于 \mathbf{ab} 端在该频率时短路，则：

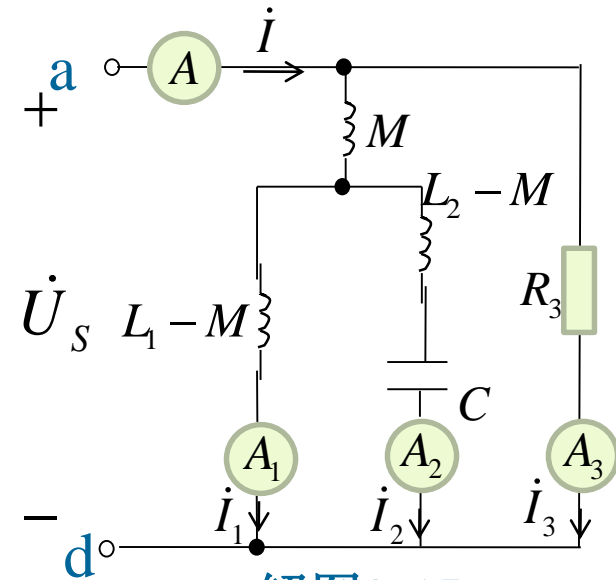
$$\omega_2 = 3 \times 314 = \frac{1}{\sqrt{L_1 C}} \Rightarrow L_1 = 0.12 \text{ H}$$

发生并联谐振，即**M**上电流为**0**，
则表 A_1 和 A_2 所在二支路电流大小相
等，则有：

$$I_2 = I_1 = \frac{U_S}{\omega(L_1 - M)}$$

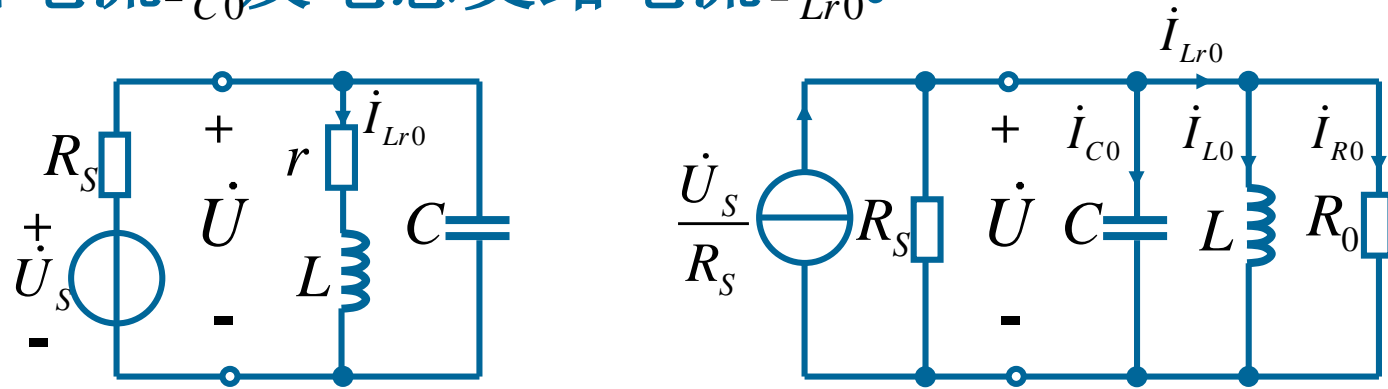
$$= \frac{500}{10^4 \times (40 - 10) \times 10^{-3}} \approx 1.67 \text{ A}$$

故，表 A_1 和 A_2 的读数为**1.67A**，而表 A_3 和表 A 的读
数为**1A**。



解图9-15

9-10 题图9-10所示并联谐振电路, $L = 0.1mH$, $C = 100pF$, $r = 10\Omega$, $R_S = 100k\Omega$, $\dot{U}_S = 2\angle 0^\circ$, 试求(1)谐振角频率 ω_0 ; (2)端电压 \dot{U} ; (3)整个电路的品质因数 Q' ; (4)谐振时电容支路电流 \dot{i}_{C0} 及电感支路电流 \dot{i}_{Lr0} 。

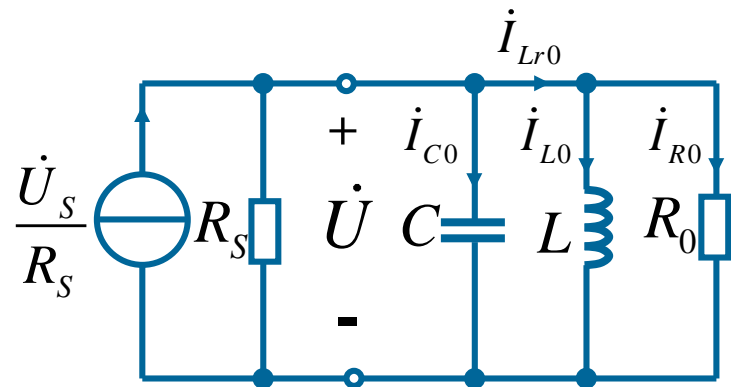


解: 这是一个实际的并联谐振电路, 其等效电路如右图所示; 且: $R_0 = \frac{L}{Cr} = 100k\Omega$

(1) $\omega_0 = \frac{1}{\sqrt{LC}} = 10^7 \text{ rad/s}$

(2) $\dot{U} = \frac{\dot{U}_S}{R_S} \times (R_S // R_0) = 1\angle 0^\circ \text{ V}$

(3) $Q' = \frac{R_S C}{G} = \omega_0 CR = \omega_0 C(R_S // R_0) = 50$



(4) $\dot{I}_{C0} = jQ' \cdot \frac{\dot{U}_s}{R_s} = 1\angle 90^\circ \text{ mA}$ (或 $\dot{I}_{C0} = j\omega_0 C\dot{U}$)

$$\dot{I}_{L0} = -\dot{I}_{C0} = 1\angle -90^\circ \text{ mA}$$

$$\dot{I}_{R0} = \frac{\dot{U}_s}{R_s} \cdot \frac{R_s}{R_s + R_0} = 0.01\angle 0^\circ \text{ mA}$$

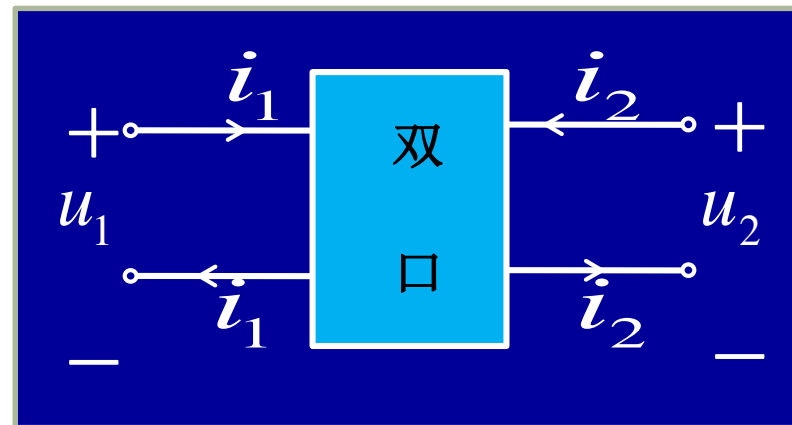
$$\therefore \dot{I}_{Lr0} = \dot{I}_{L0} + \dot{I}_{R0} = 0.01\angle 0^\circ + 1\angle -90^\circ = 0.01 - j1 \text{ mA}$$

第11章 二端口网络的方程与参数

一、Z参数

将端口电流 i_1 、 i_2 作为自变量。

$$\begin{cases} \dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ \dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2 \end{cases} \quad Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$



$$Z_{11} = \frac{\dot{U}_1}{\dot{I}_1} \Big|_{i_2=0} \text{ 输出端口开路时的输入阻抗}$$

$$Z_{12} = \frac{\dot{U}_1}{\dot{I}_2} \Big|_{i_1=0} \text{ 输入端口开路时的转移阻抗}$$

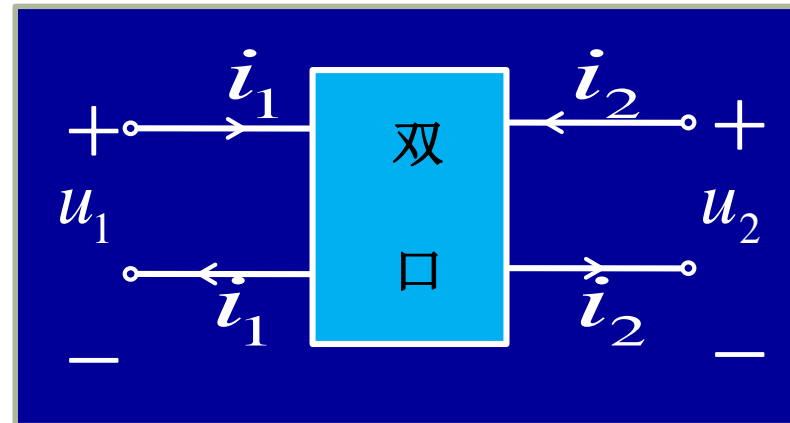
$$Z_{21} = \frac{\dot{U}_2}{\dot{I}_1} \Big|_{i_2=0} \text{ 输出端口开路时的转移阻抗}$$

$$Z_{22} = \frac{\dot{U}_2}{\dot{I}_2} \Big|_{i_1=0} \text{ 输入端口开路时的输出阻抗}$$

二、Y参数

将端口电压 u_1 、 u_2 作为自变量。

$$\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases} \quad Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$



$$Y_{11} = \left. \frac{\dot{I}_1}{\dot{U}_1} \right|_{\dot{U}_2=0} \text{ 输出端口短路时的输入导纳}$$

$$Y_{12} = \left. \frac{\dot{I}_1}{\dot{U}_2} \right|_{\dot{U}_1=0} \text{ 输入端口短路时的转移导纳}$$

$$Y_{21} = \left. \frac{\dot{I}_2}{\dot{U}_1} \right|_{\dot{U}_2=0} \text{ 输出端口短路时的转移导纳}$$

$$Y_{22} = \left. \frac{\dot{I}_2}{\dot{U}_2} \right|_{\dot{U}_1=0} \text{ 输入端口短路时的输出导纳}$$

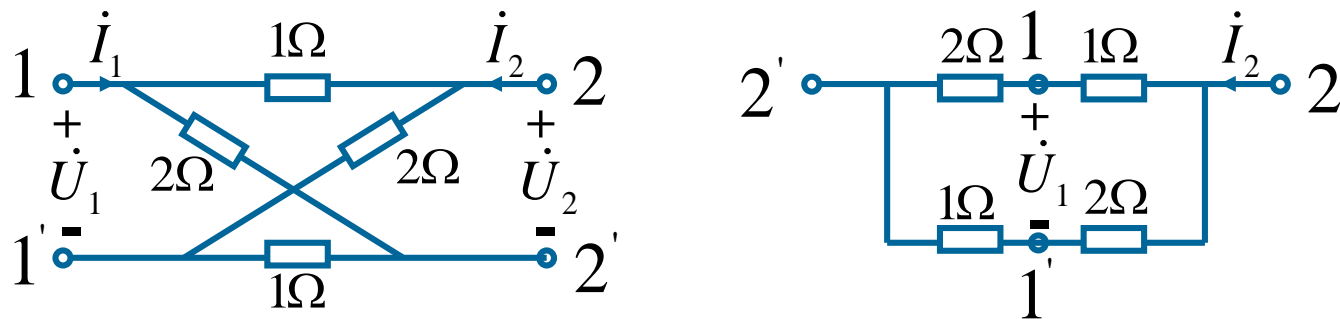
计算方法：

- **直接应用定义**，将电路输入、输出端口开、短路后进行计算。
- **列写网络方程(节点方程、网孔方程)**，消去方程中的非端口变量得到网络的参数方程，其系数即为网络参数。

线性无源网络： $Z_{12} = Z_{21}$ ， $Y_{12} = Y_{21}$

对称电路： $Z_{11} = Z_{22}$ ， $Y_{11} = Y_{22}$

11-1(a) 求题图得11-1所示二端网络的Z参数。



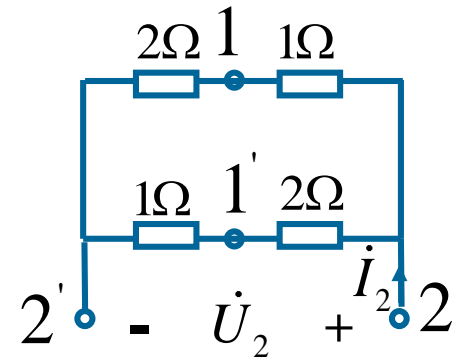
解：利用Z参数的定义求解。设端口电压、电流如图，则有：

$$Z_{11} = \left. \frac{\dot{U}_1}{\dot{I}_1} \right|_{\dot{I}_2=0} = (1+2) // (2+1) = 1.5\Omega$$

$\dot{I}_1=0$ 时等效电路如右图，则有：

$$Z_{12} = \left. \frac{\dot{U}_1}{\dot{I}_2} \right|_{\dot{I}_1=0} = \frac{\frac{\dot{I}_2}{2} \times 2 - \frac{\dot{I}_2}{2} \times 1}{\dot{I}_2} = 0.5\Omega$$

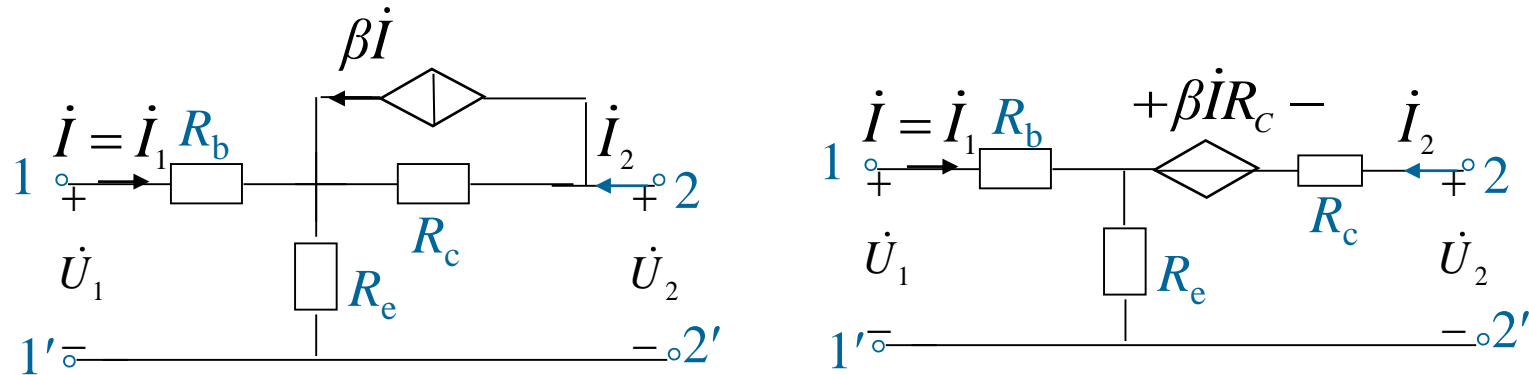
因原网络是线性无源二端网络，有： $Z_{21} = Z_{12} = 0.5\Omega$



$$Z_{22} = \frac{\dot{U}_2}{\dot{I}_2} \Big|_{i_1=0} = (1+2) // (2+1) = 1.5\Omega$$

$$\therefore Z = \begin{bmatrix} 1.5 & 0.5 \\ 0.5 & 1.5 \end{bmatrix} \Omega$$

11-1 (b) 求题图11-1所示二端口网络的Z参数。

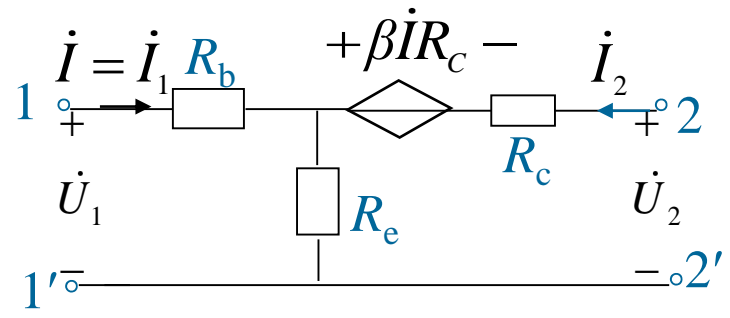
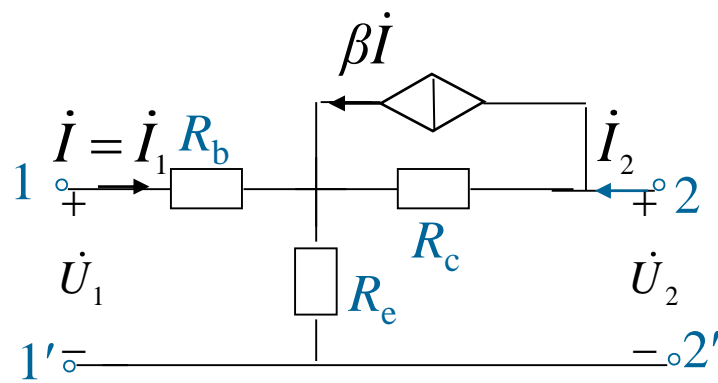


解：方法一：利用Z参数的物理意义求解。

设图所示二端口网络端子上电压、电流参考方向，则

$$Z_{11} = \left. \frac{\dot{U}_1}{\dot{I}_1} \right|_{\dot{I}_2=0} = \frac{(R_b + R_e)\dot{I}_1}{\dot{I}_1} \Big|_{\dot{I}_2=0} = R_b + R_e$$

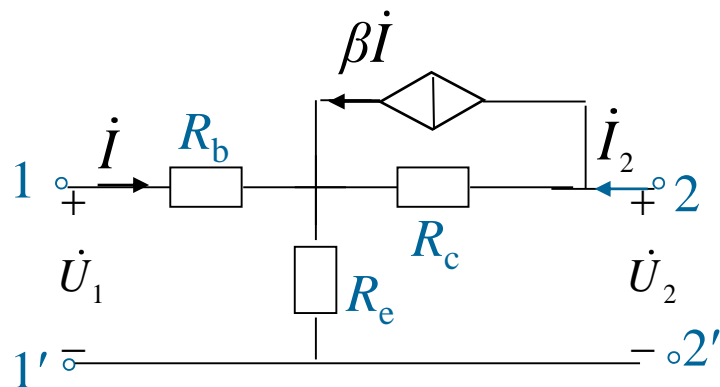
$$Z_{12} = \left. \frac{\dot{U}_1}{\dot{I}_2} \right|_{\dot{I}_1=0} = \frac{R_e \dot{I}_2}{\dot{I}_2} \Big|_{\dot{I}_1=0} = R_e$$



$$Z_{21} = \left. \frac{\dot{U}_2}{\dot{I}_1} \right|_{i_2=0} = \left. \frac{-\beta \dot{I}_1 R_c + \dot{I}_1 R_e}{\dot{I}_1} \right|_{i_2=0} = R_e - \beta R_c$$

$$Z_{22} = \left. \frac{\dot{U}_2}{\dot{I}_2} \right|_{i_1=0} = \left. \frac{(R_c + R_e) \dot{I}_2}{\dot{I}_2} \right|_{i_1=0} = R_c + R_e$$

方法二：利用参数方程求解。

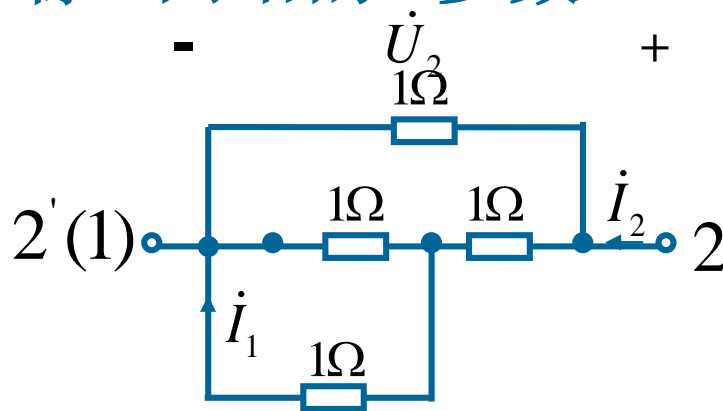
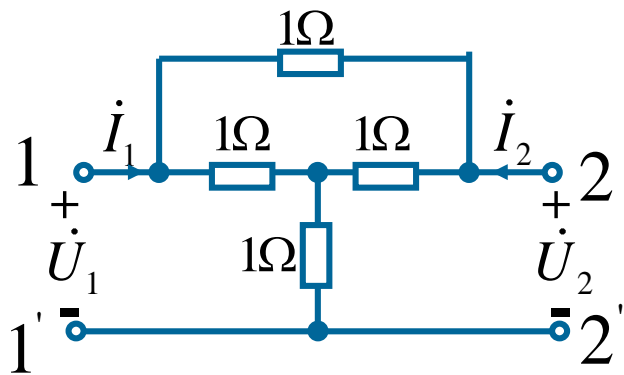


$$\begin{cases} \dot{U}_1 = R_b \dot{I} + R_e (\dot{I} + \dot{I}_2) \\ \dot{U}_2 = R_c (\dot{I}_2 - \beta \dot{I}) + R_e (\dot{I} + \dot{I}_2) \end{cases}$$

$$\begin{cases} \dot{U}_1 = (R_b + R_e) \dot{I} + R_e \dot{I}_2 \\ \dot{U}_2 = (R_e - \beta R_c) \dot{I} + (R_e + R_c) \dot{I}_2 \end{cases}$$

$$\therefore \mathbf{Z} = \begin{bmatrix} R_b + R_e & R_e \\ R_e - \beta R_c & R_e + R_c \end{bmatrix} \boldsymbol{\Omega}$$

11-2(a) 求题图得11-2所示二端口网络的Y参数。



解：设端口电压、电流如图，则有：

$\dot{U}_2 = 0$ ，相当于2和2'端短路，故：

$$Y_{11} = \left. \frac{\dot{I}_1}{\dot{U}_1} \right|_{\dot{U}_2=0} = \frac{1}{1 // (1 + 1 // 1)} = \frac{5}{3} S$$

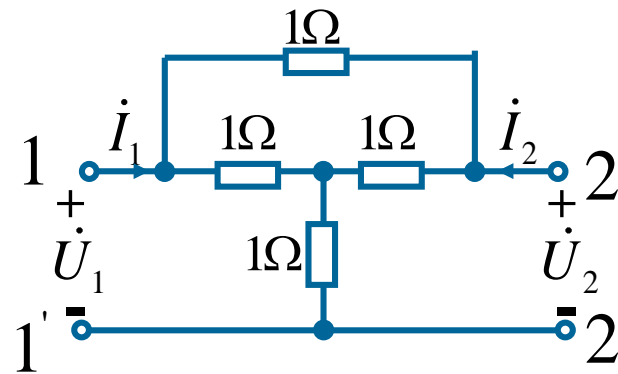
$$Y_{22} = Y_{11} = \frac{5}{3} S, Y_{21} = Y_{12} = -\frac{4}{3} S$$

$\dot{U}_1 = 0$ ，1和1'端短路，等效电路如右图：

$$Y_{12} = \left. \frac{\dot{I}_1}{\dot{U}_2} \right|_{\dot{U}_1=0} = -\frac{\dot{U}_2 \times 1 + \frac{\dot{U}_2}{1+1//1} \cdot \frac{1}{2}}{\dot{U}_2} = -\frac{4}{3} S$$

$$\therefore Y = \begin{bmatrix} \frac{5}{3} & -\frac{4}{3} \\ -\frac{4}{3} & \frac{5}{3} \end{bmatrix} S$$

另解：列节点方程



$$\begin{cases} 2\dot{U}_1 - \dot{U}_x - \dot{U}_2 = \dot{I}_1 \\ 2\dot{U}_2 - \dot{U}_x - \dot{U}_1 = \dot{I}_2 \\ 3\dot{U}_x - \dot{U}_1 - \dot{U}_2 = 0 \end{cases} \Rightarrow \dot{U}_x = \frac{\dot{U}_1 + \dot{U}_2}{3}$$

$$\begin{cases} \frac{5}{3}\dot{U}_1 - \frac{4}{3}\dot{U}_2 = \dot{I}_1 \\ -\frac{4}{3}\dot{U}_1 + \frac{5}{3}\dot{U}_2 = \dot{I}_2 \end{cases}$$

$$\therefore Y = \begin{bmatrix} \frac{5}{3} & -\frac{4}{3} \\ -\frac{4}{3} & \frac{5}{3} \end{bmatrix} S$$

11-2 (b) 求题图11-2所示二端口网络的Y参数。

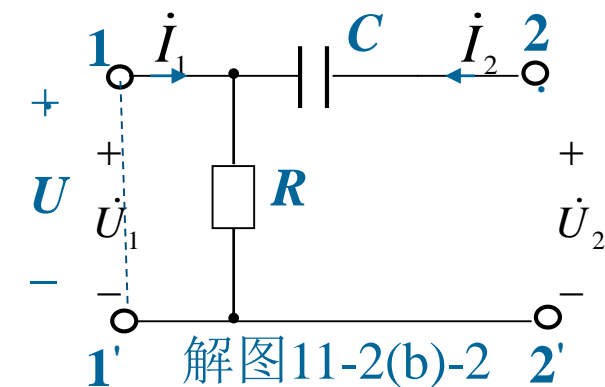
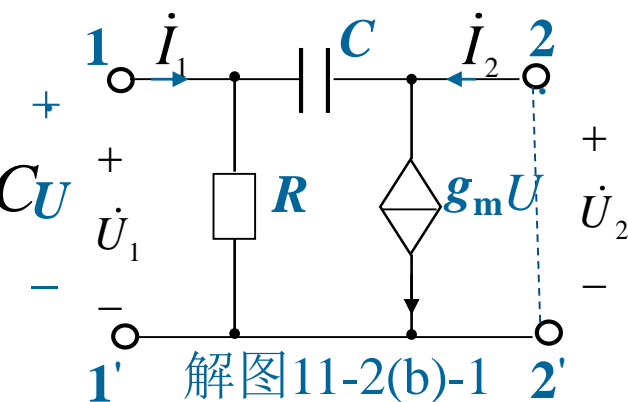
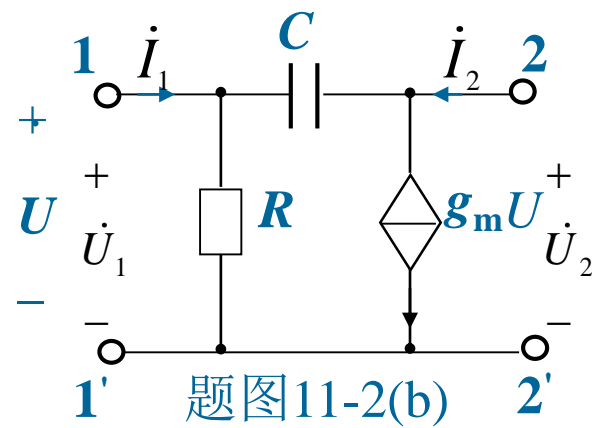
解：方法一：利用Y参数的物理意义求解。

$\dot{U}_2=0$ 时如解图11-2(b)-1 所示：

$$Y_{11} = \left. \frac{\dot{I}_1}{\dot{U}_1} \right|_{\dot{U}_2=0} = \frac{\dot{U}_1 + j\omega C \dot{U}_1}{\dot{U}_1} \Bigg|_{\dot{U}_2=0} = \frac{1}{R} + j\omega C$$

$\dot{U}_1 = 0$ 时如解图11-2(b)-2 所示：

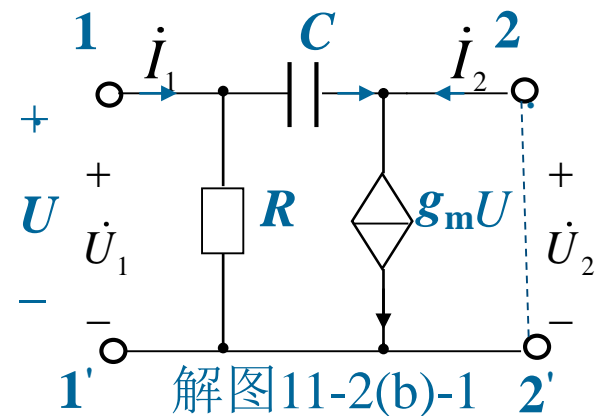
$$Y_{12} = \left. \frac{\dot{I}_1}{\dot{U}_2} \right|_{\dot{U}_1=0} = -j\omega C$$



$\dot{U}_2=0$ 时如解图11-2(b)-1 所示:

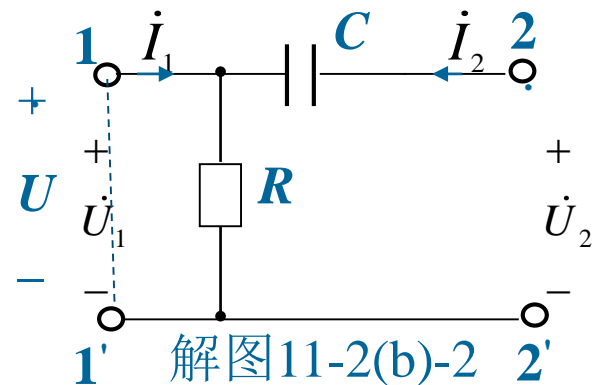
$$Y_{21} = \left. \frac{\dot{I}_2}{\dot{U}_1} \right|_{\dot{U}_2=0} = \left. \frac{g_m \dot{U}_1 - j\omega C \dot{U}_1}{\dot{U}_1} \right|_{\dot{U}_2=0}$$

$$= g_m - j\omega C$$



$\dot{U}_1=0$ 时如解图11-2(b)-2 所示:

$$Y_{22} = \left. \frac{\dot{I}_2}{\dot{U}_2} \right|_{\dot{U}_1=0} = j\omega C$$

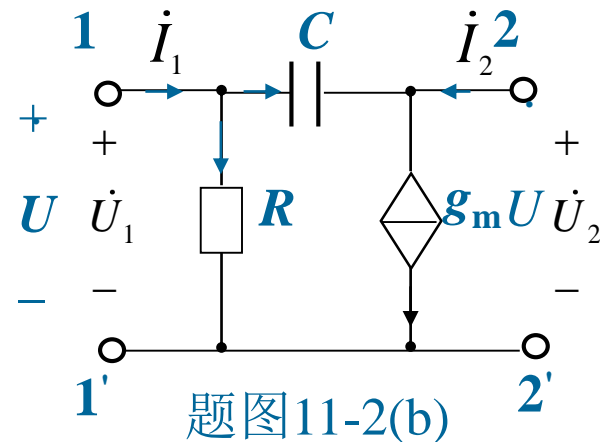


方法二：利用参数方程求解。

$$\begin{cases} \dot{I}_1 = \frac{\dot{U}}{R} + j\omega C(\dot{U}_1 - \dot{U}_2) \\ \dot{I}_2 = g_m \dot{U}_1 - j\omega C(\dot{U}_1 - \dot{U}_2) \end{cases}$$

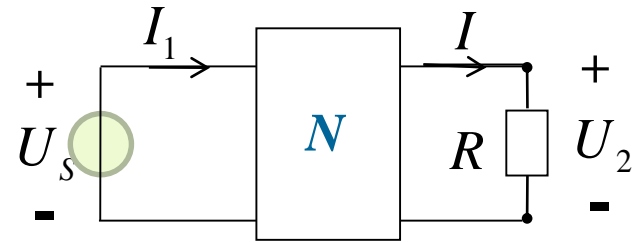
$$\begin{cases} \dot{I}_1 = \left(\frac{1}{R} + j\omega C\right)\dot{U}_1 + (-j\omega C)\dot{U}_2 \\ \dot{I}_2 = (g_m - j\omega C)\dot{U}_1 + j\omega C\dot{U}_2 \end{cases}$$

$$Y = \begin{bmatrix} \frac{1}{R} + j\omega C & -j\omega C \\ g_m - j\omega C & j\omega C \end{bmatrix} S$$



11-4 在题图11-4所示电路中，已知N为线性电阻网络，且当 $U_S = 8V, R = 3\Omega$ 时， $I = 0.5A$ ； $U_S = 18V, R = 4\Omega$ 时， $I = 1A$ 。试求当 $U_S = 25V, R = 6\Omega$ 时： $I = ?$

解：因为已知两个端口电压，而要求端口电流，则可用Y参数方程：



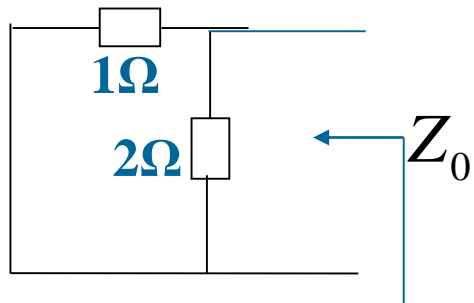
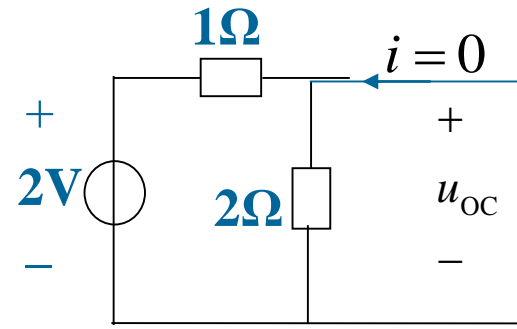
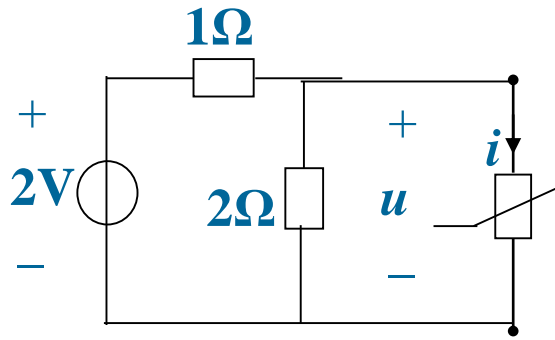
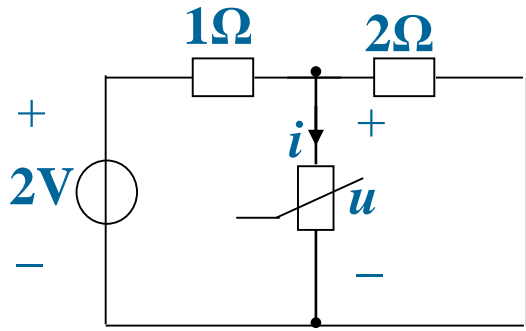
$$\begin{cases} I_1 = Y_{11}U_S + Y_{12}U_2 \\ -I = Y_{21}U_S + Y_{22}U_2 \end{cases} \quad \text{而：} \quad U_2 = RI$$

将已知条件代入第二个式子，则有：

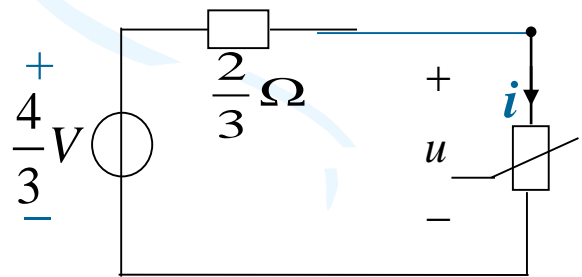
$$\begin{cases} -0.5 = 8Y_{21} + 1.5Y_{22} \\ -1 = 18Y_{21} + 4Y_{22} \end{cases} \Rightarrow \begin{cases} Y_{21} = -0.1S \\ Y_{22} = 0.2S \end{cases}$$

故当 $U_S = 25V, R = 6\Omega$ 时，有： $I = -(Y_{21}U_S + Y_{22}U_2) \approx 1.14A$

5-2 题图5-2所示电路中，若非线性电阻的伏安关系为 $i = u + 0.13u^2$ ，试求电流 i 。



$$u_{oc} = 2 \times \frac{2}{1+2} = \frac{4}{3} \text{ V} \quad Z_0 = \frac{1 \times 2}{1+2} = \frac{2}{3} \Omega$$



$$\begin{cases} u = \frac{4}{3} - \frac{2}{3}i \\ i = u + 0.13u^2 \end{cases} \quad \begin{cases} u_1 = 0.77 \text{ V}, & i_1 = 0.845 \text{ A} \\ u_2 = -20 \text{ V}, & i_2 = 32 \text{ A} \end{cases}$$

思考题：如题图所示电路，是角频率为 ω 的正弦电压源。已知 $L_1=L_2=5\text{mH}$ ， $M=1\text{mH}$ ， $C=0.4\mu\text{F}$ ， $R=10\Omega$ ，若电流 $I=0$ ，试求电源角频率。

