第2章 电路分析中的等效变换

电阻电路分析法：

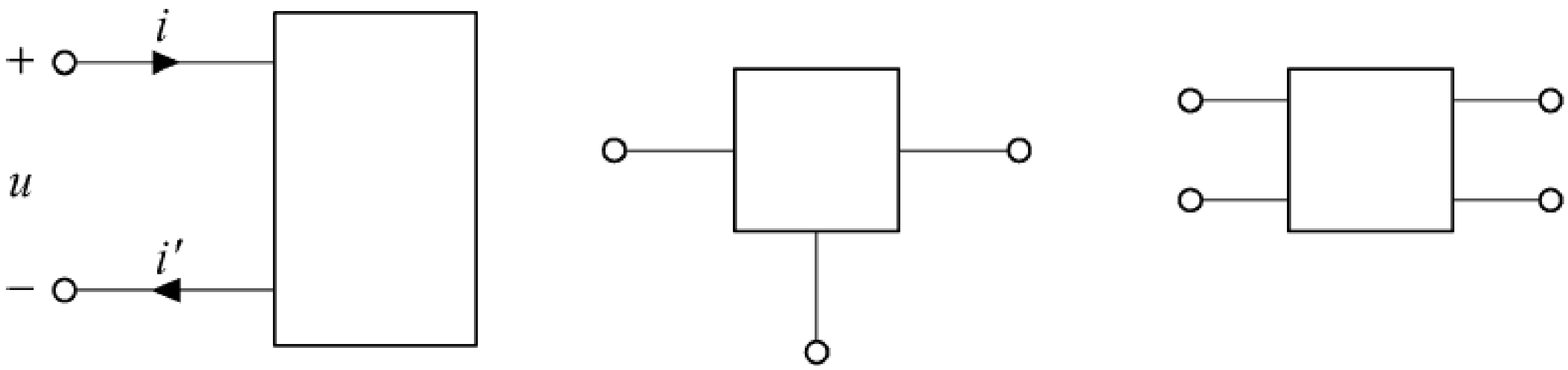
一、等效变换 —求局部响应 （第2章）

二、一般分析方法—系统化求响应（第3章）

三、网络定理 （第4章）

等效二端网络的概念

◆等效二端网络的概念



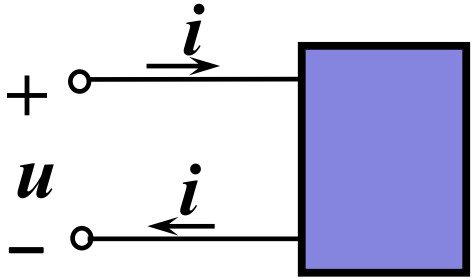
(a) (b) (c)

单口网络

双口网络

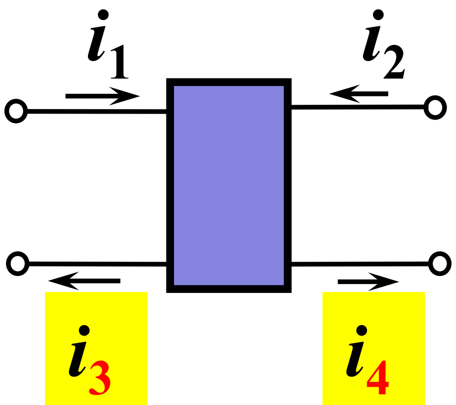
双口网络

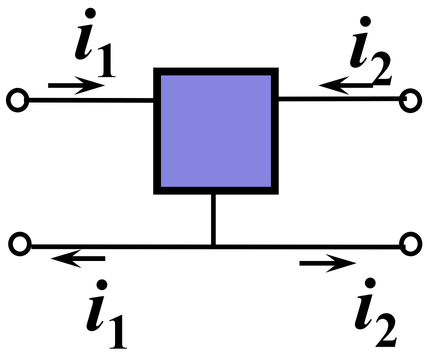
按端口数目来分类



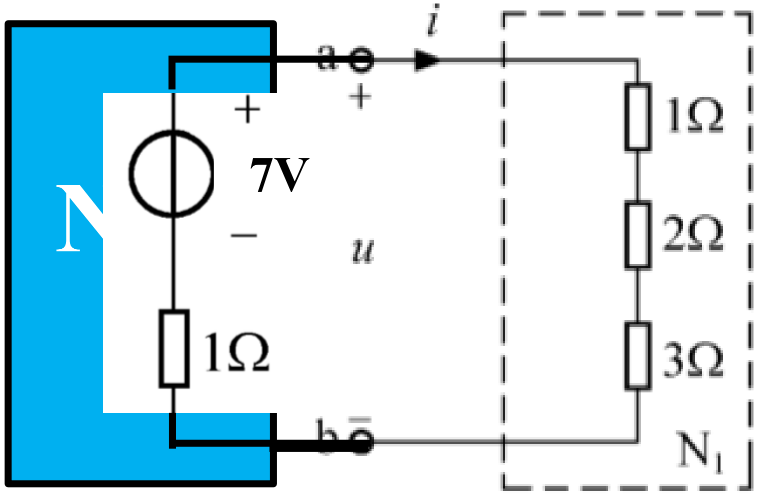
端口由一对端子构成，且满足从一个端子流入的电流等于从另一个端子流出的电流。

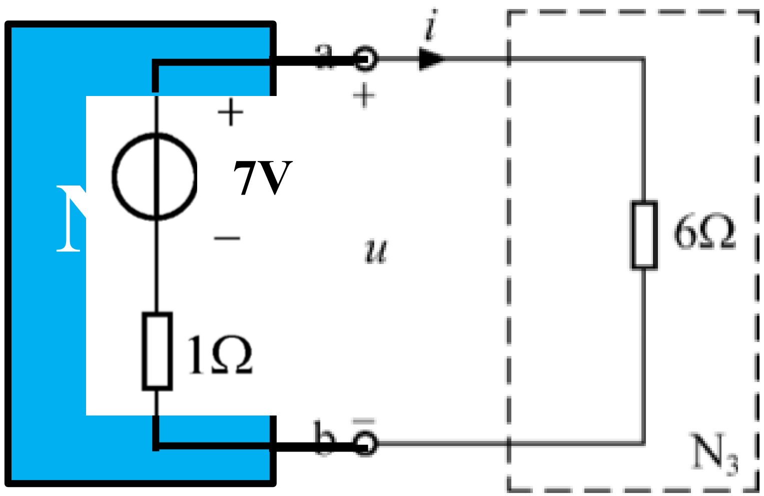
当一个电路与外部电路通过两个端口连接时称此电路为二端口网络（双口网络）。





2-1 等效二端网络





$$u = 1 \times i + 2 \times i + 3 \times i$$

$$= 6i$$

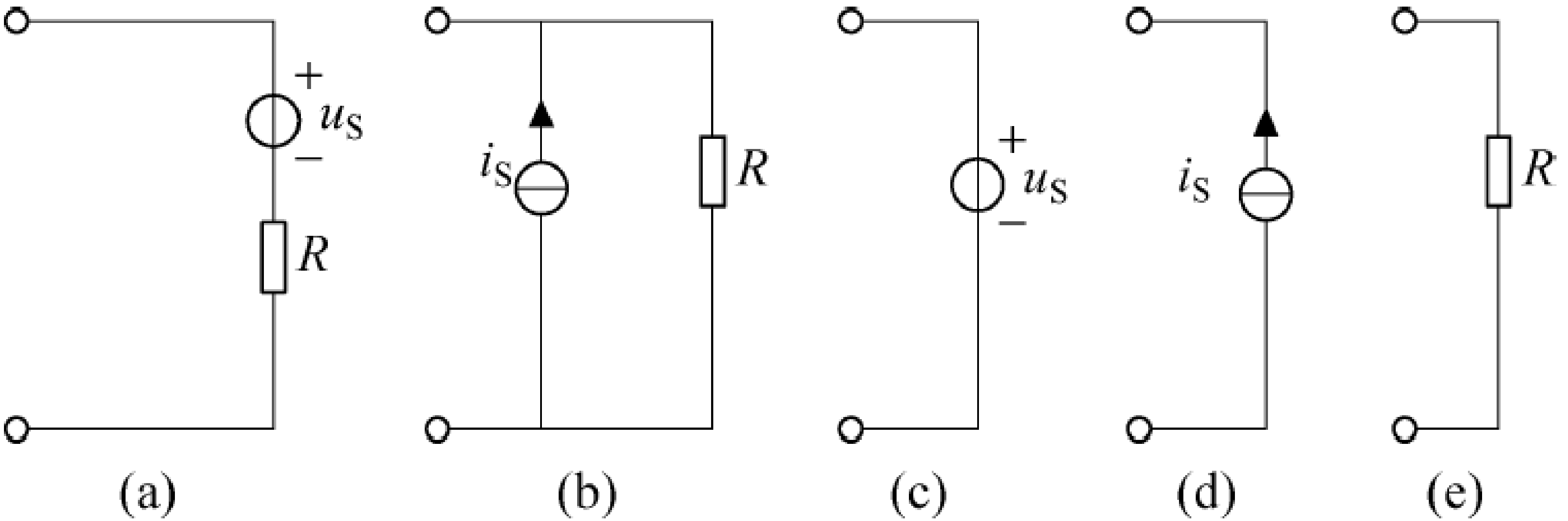
$$u = 6i$$

N_1和N_3内部结构不同，但具有相同的端口电压电流关系（VCR）,对于外电路N0作用效果相同。

等效变换：网络的一部分用结构不同但端子数和端子上VCR完全相同的另一部分来代替。替代后对余下部分来说，其作用效果完全相同，这两部分电路称等效电路。

对外等效，对内不等效。

最简二端网络



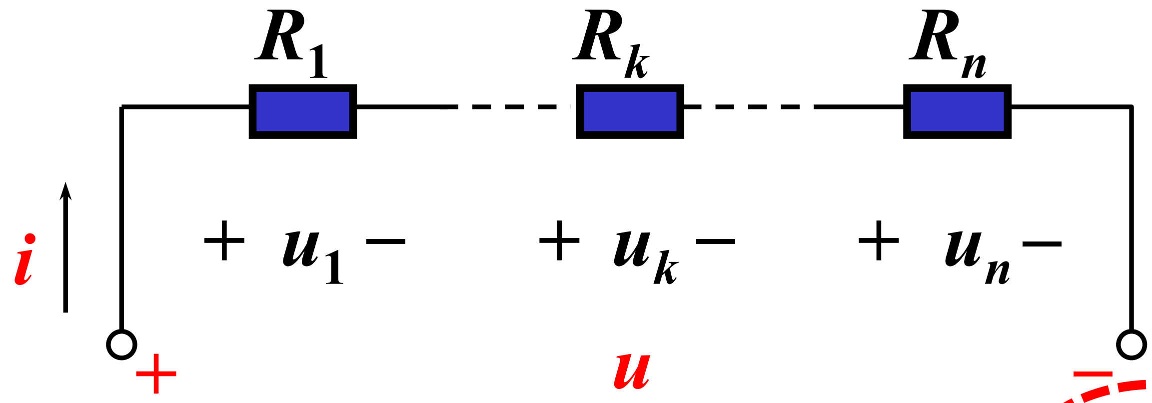
✓无源二端网络？

✓有源二端网络？

无源二端电阻网络

电阻串联

一、电路特点

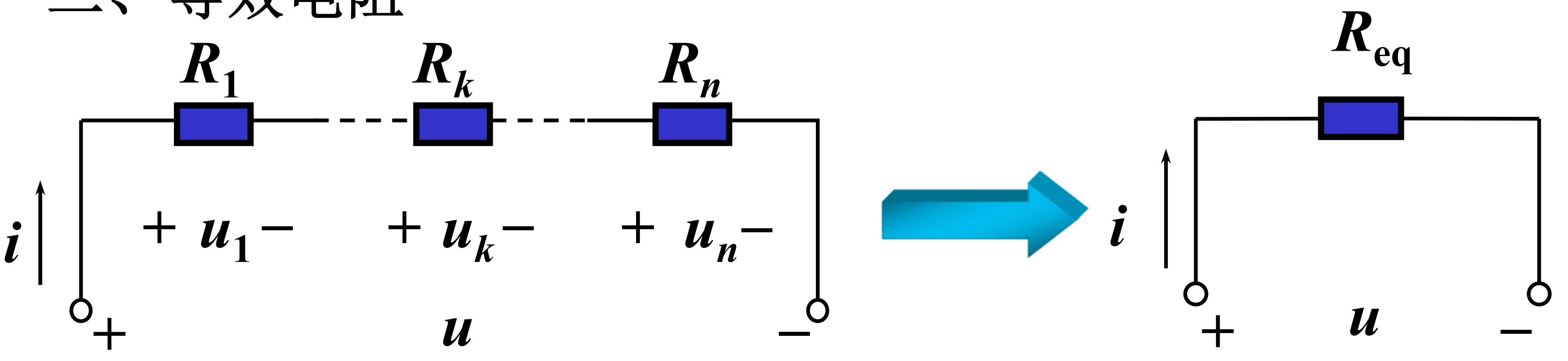


1、各电阻顺序连接，流过同一电流（KCL）；

2、总电压等于各串联电阻上的电压之和（KVL）；

$$u = u_1 + \ldots + u_k + \ldots + u_n$$

二、等效电阻



$$R_{eq}=(R_1+R_2+\ldots+R_n)=\sum R_k$$

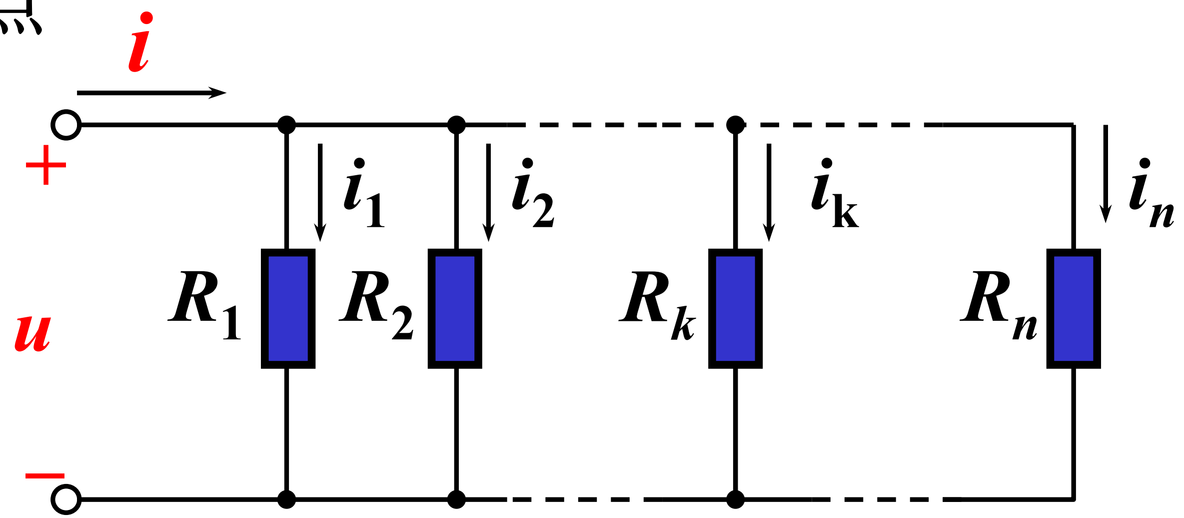
$$u_k=\frac{R_k}{R_{eq}}u$$

$$p_k=u_k\cdot i=R_k\cdot i^2$$

$$p_{总}=u\cdot i=R_{eq}\cdot i^2$$

电阻并联

一、电路特点

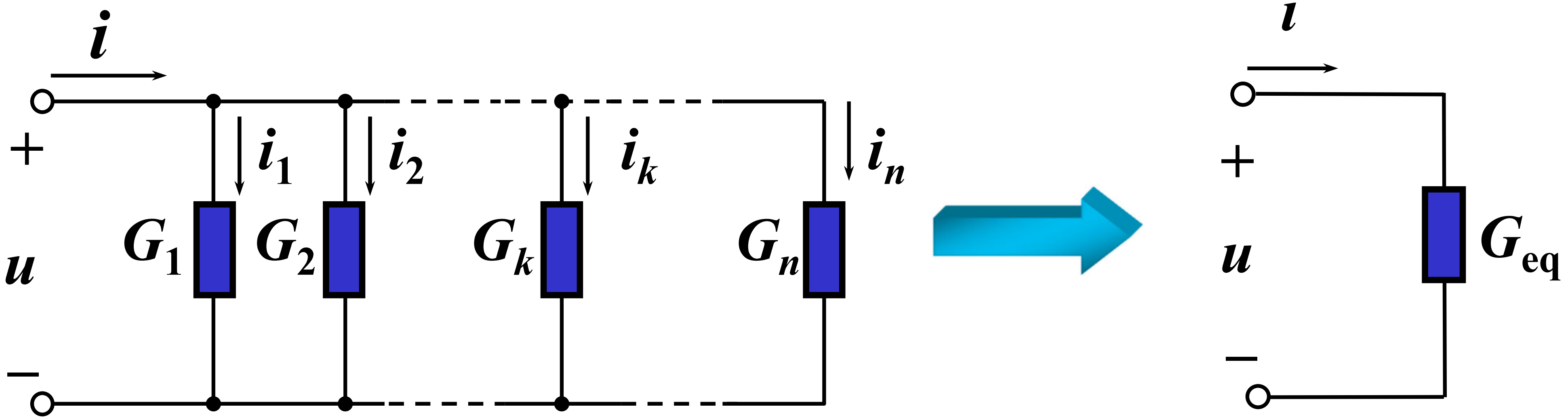


1、各电阻两端分别接在一起，端电压为同一电压（KVL）；

2、总电流等于流过各并联电阻的电流之和（KCL）：

$$i = i_1 + i_2 + \cdots + i_k + \cdots + i_n$$

二、等效电导 $G_k = 1/R_k$ （$k = 1, 2, \cdots, n$）单位：西门子S



$$G_{eq}=G_1+G_2+\cdots+G_k+\cdots+G_n=\sum G_k=\sum 1/R_k$$

$$i_k=\frac{G_k}{G_{eq}}i$$

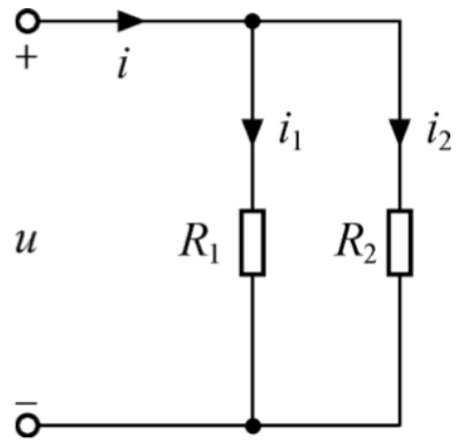
$$p_k=u\cdot i_k=u^2G_k$$

$$p_{总}=u\cdot i=u^2G_{eq}$$

$$R_{eq}=\frac{R_1R_2}{R_1+R_2}$$

$$i_1=\frac{G_1}{G_1+G_2}\cdot i=\frac{R_2}{R_1+R_2}i$$

$$i_2=\frac{G_2}{G_1+G_2}\cdot i=\frac{R_1}{R_1+R_2}i$$



若 $R_1=R_2=R$ 则 $R_{eq}=\frac{R}{2}$

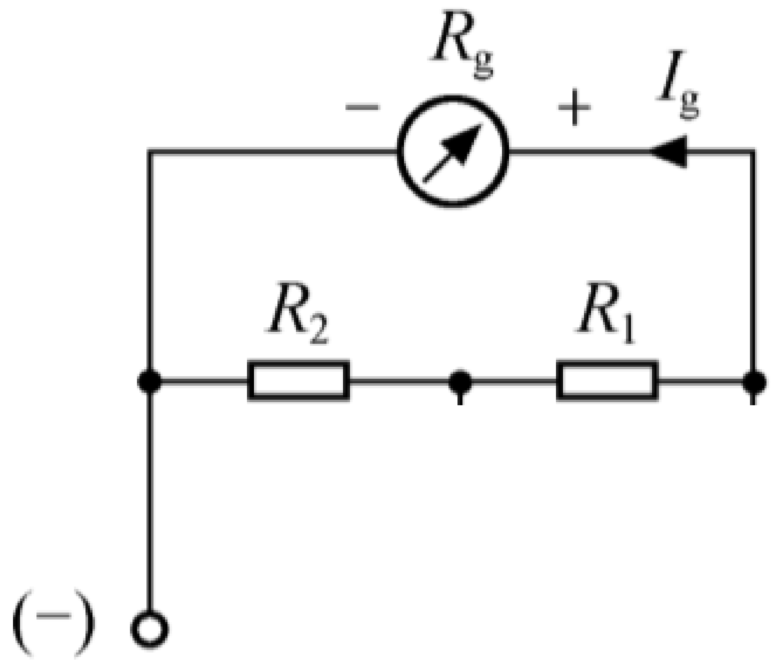
若n个R并联，则 $R_{eq}=\frac{R}{n}$ $i_k=\frac{i}{n}$

例1：I g = 50 u A , R g = 2 K Ω 。欲把量程扩大为 5 m A和 50 m A，求R1和R2。

$$Ig = \frac{R_1 + R_2}{R_1 + R_2 + Rg} I_1$$

$$Ig = \frac{R_2}{R_1 + R_2 + Rg} I_2$$

$I_1 < I_2$



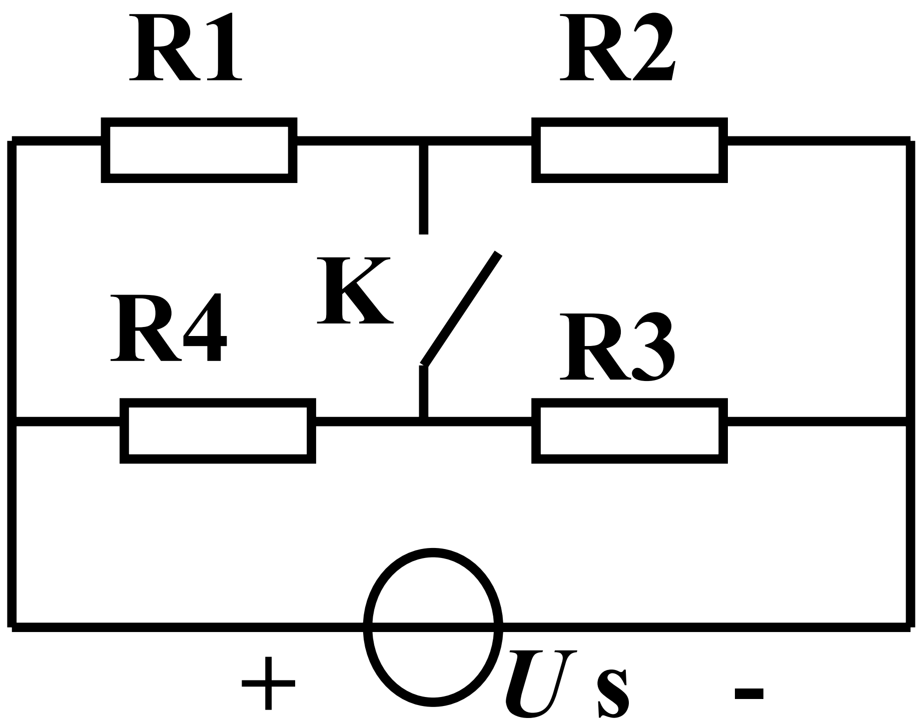
I_1=5 m A， I_2=50 mA

代入参数，得 $R_1 = 18\Omega, R_2 = 2\Omega$

例2： R1=40 Ω ， R2=30 Ω ， R3=20 Ω ，R4=10 Ω ，U_s = 60V

（1） K打开时，求开关两端电压

（2） K闭合时，求流经开关的电流



R1=40 ， R2=30 ， R3=20 , R4=10 , $U_s = 60\text{V}$

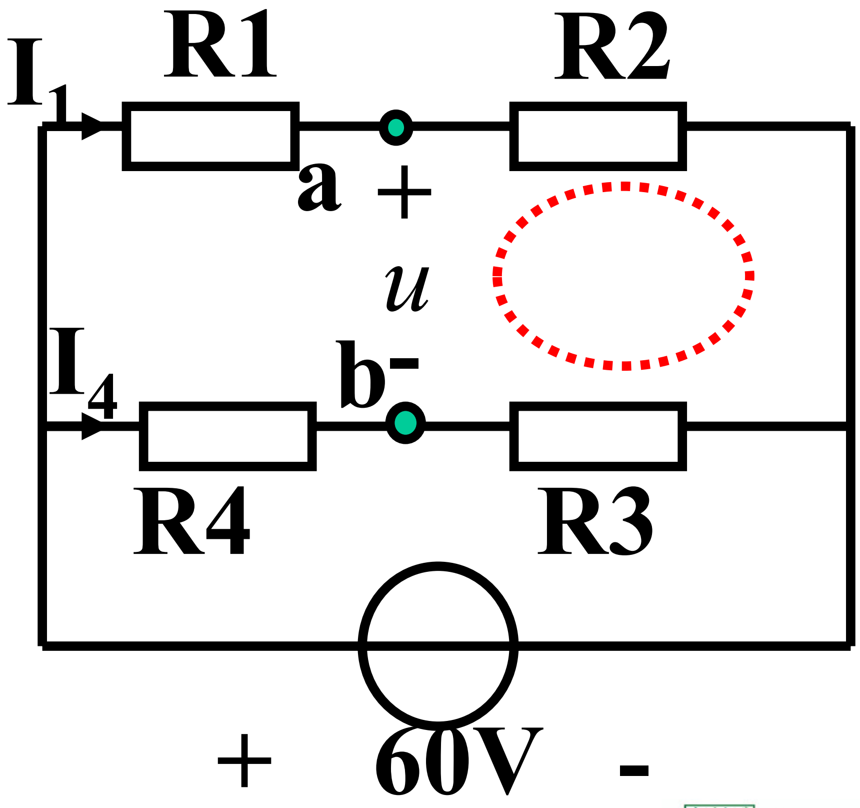
解：(1)各支路电流如图，则

$$I_1 = \frac{u_S}{R_1 + R_2} = \frac{6}{7}\text{A}$$

$$I_4 = \frac{u_S}{R_3 + R_4} = 2\text{A}$$

由假想回路，得

$$u = I_1 R_2 - I_4 R_3 = -\frac{100}{7}\text{V}$$



R1=40 ， R2=30 ， R3=20 ，R4=10 ，$U_s=60\text{V}$

(2)

$$I_s=\frac{u_s}{R_1//R_4+R_2//R_3}=3\text{A}$$

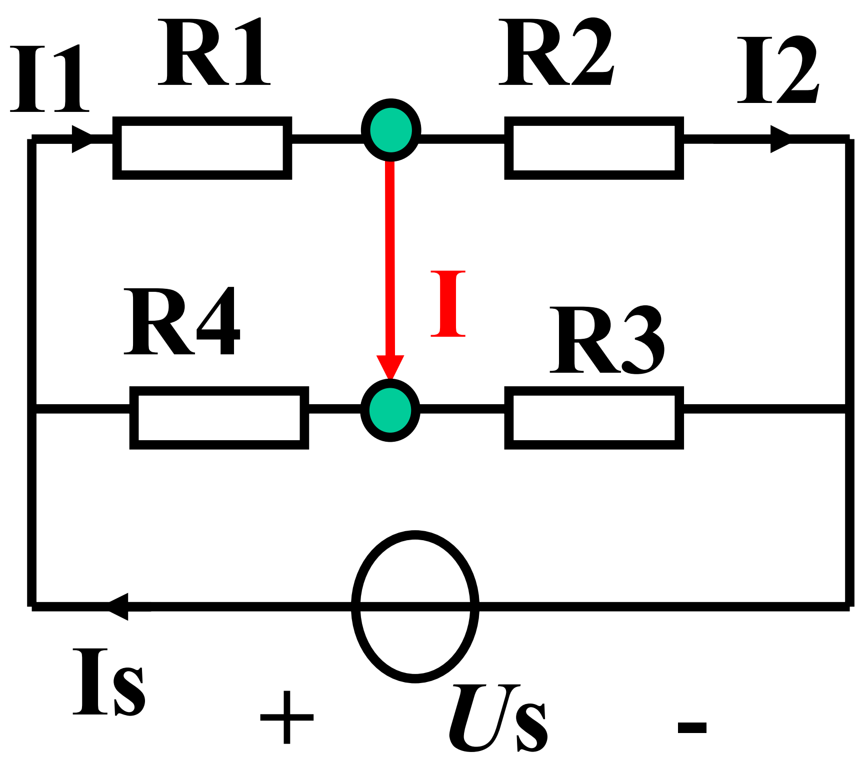
$$I_1=\frac{R_4}{R_1+R_4}I_s$$

$$=0.6\text{A}$$

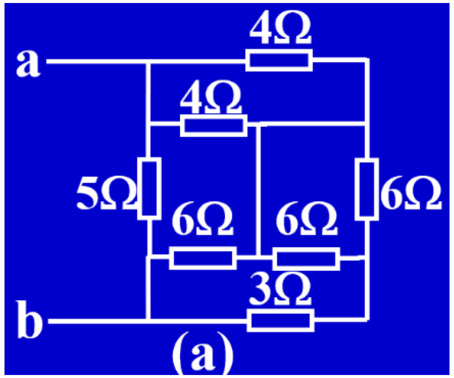
$$I_2=\frac{R_3}{R_2+R_3}I_s$$

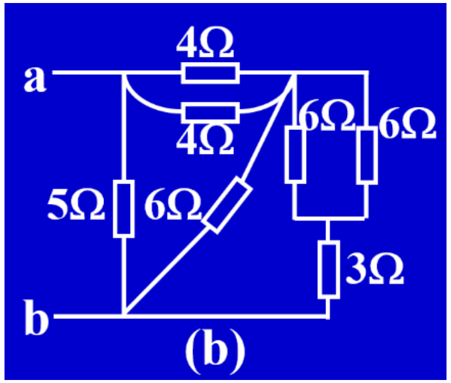
$$=1.2\text{A}$$

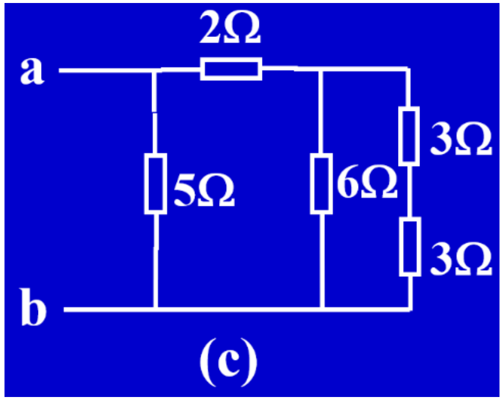
$$I=I_1-I_2=-0.6\text{A}$$

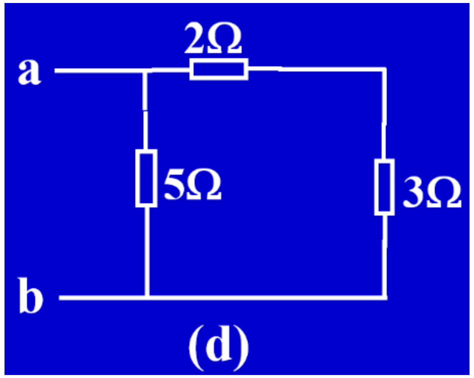


例3： 试求图 (a)所示电路a、b端的等效电阻R_{ab}。









$$R_{ab}=5//(2+3)=2.5\ \Omega$$

桥式电路（电桥）

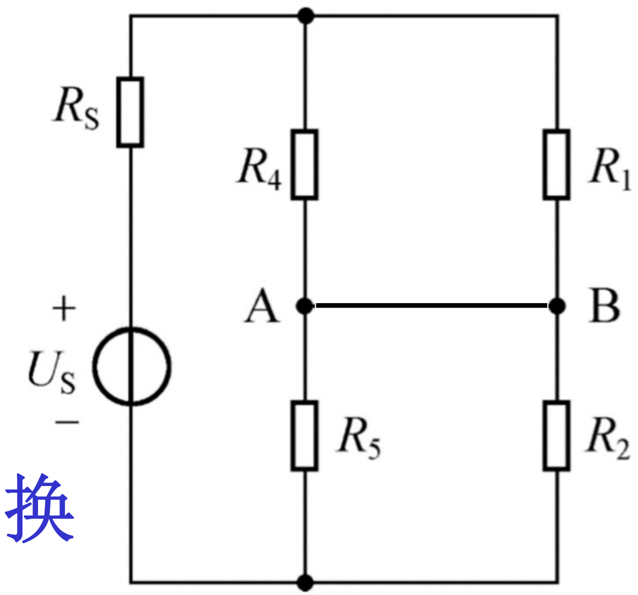
平衡条件：$R_1R_5=R_2R_4$

此时，中间桥接电阻R_3支路可视作：

1.断开 ⟶ $I=0$

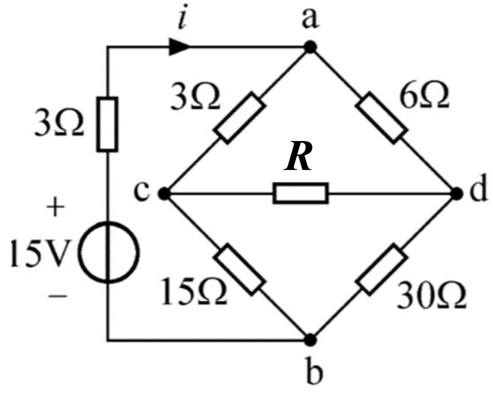
2.短路 ⟶ $U_{AB}=0$

} 等效变换



并不会影响其他支路（外电路）的响应。

例4：求i



(a)

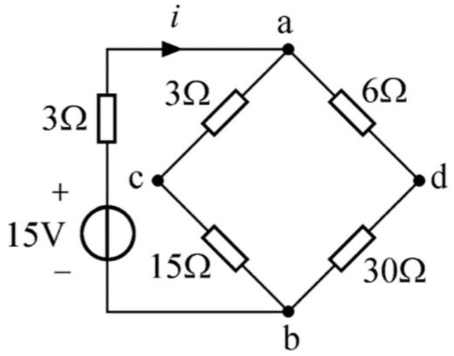
解：3*30=6*15

电桥平衡，(1) R上电流为0。R可看作开路。i_{cd}=0,如图b所示。

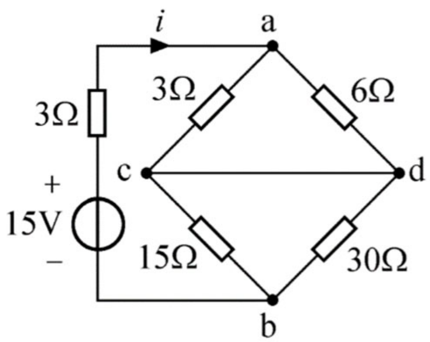
$$R_{ab}=(3+15)//(6+30)=12\Omega$$

2) R上电压为0。R可看作短路。u_{cd}=0,如图c所示。

$$R_{ab}=(3//6)+(15//30)=12\Omega$$



(b)



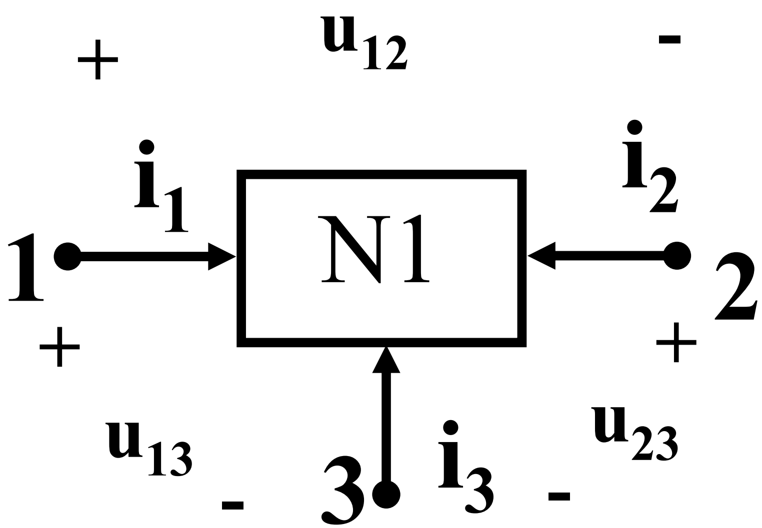
(c)

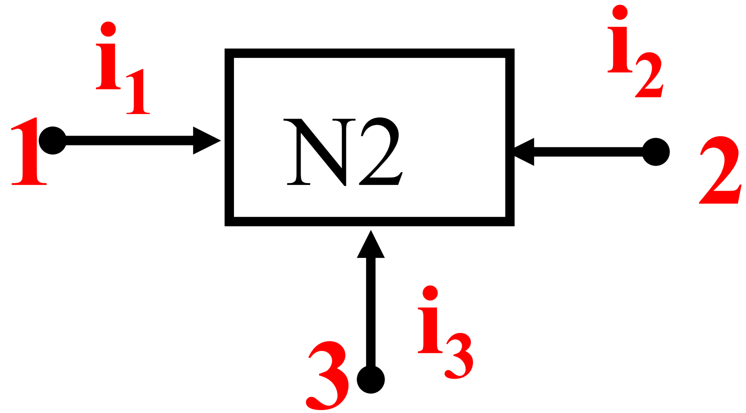
因此，两种方法都可得

$$i=\frac{15}{3+12}=1\text{A}$$

电阻星形连接与三角形连接的等效变换

三端网络的等效

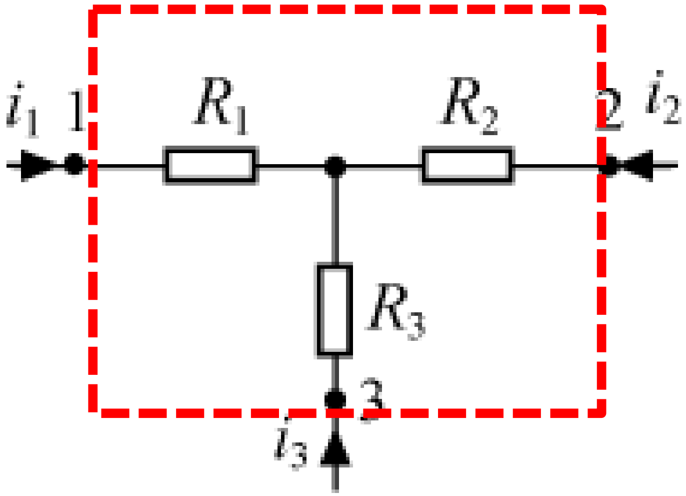




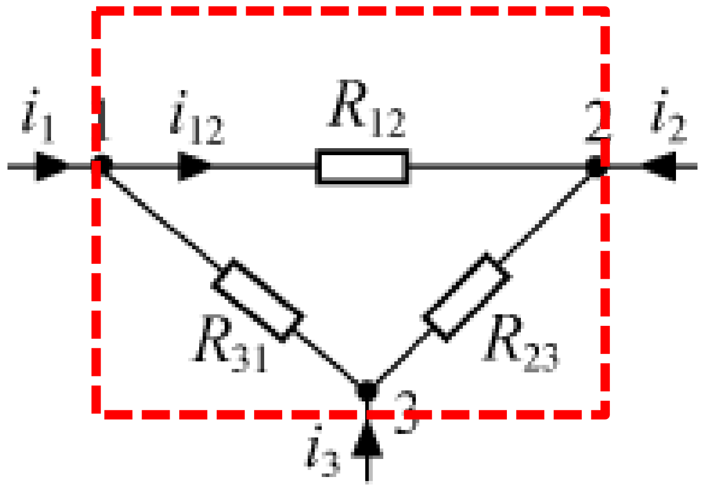
端子只有2个电流独立； 2个电压独立。

若N1与N2的 i_1，i_2，u_{13}，u_{23}间的关系完全相同，则N1与N2等效

电路符号



(a)

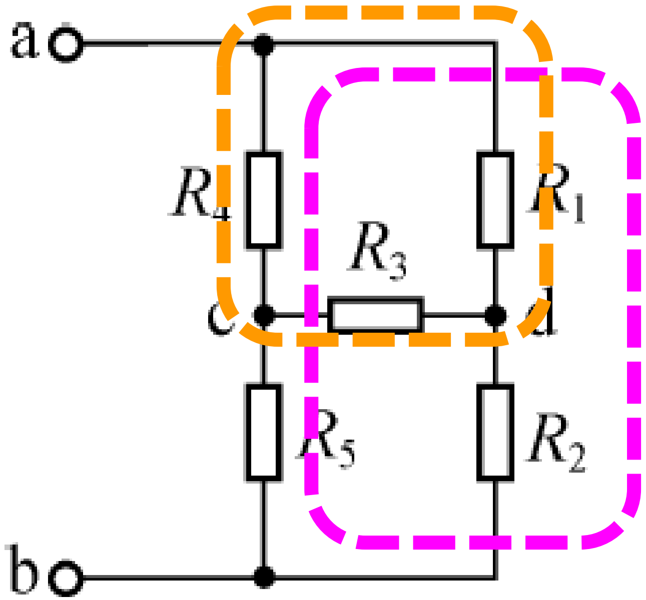


(b)

星形电阻连接

三角形电阻连接

桥式电路的等效变换

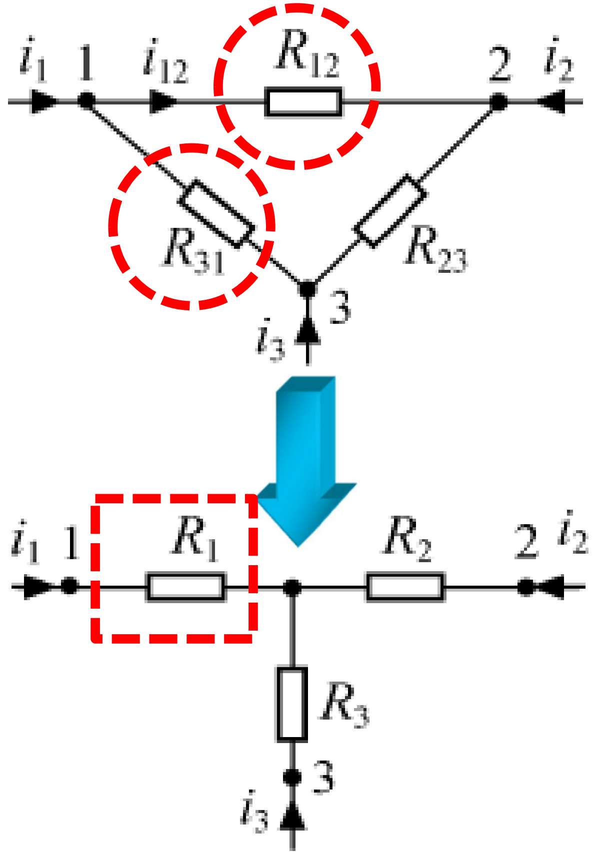


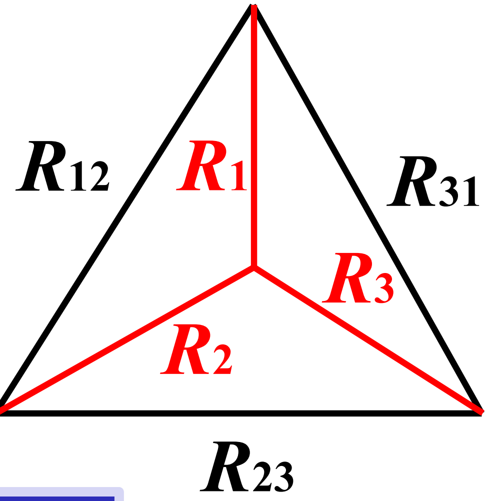
变换公式一

$$R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{31}+R_{23}}$$

$$R_2 = \frac{R_{12}R_{23}}{R_{12}+R_{31}+R_{23}}$$

$$R_3 = \frac{R_{23}R_{31}}{R_{12}+R_{31}+R_{23}}$$





三角形电阻 → 星形电阻连接

变换公式一

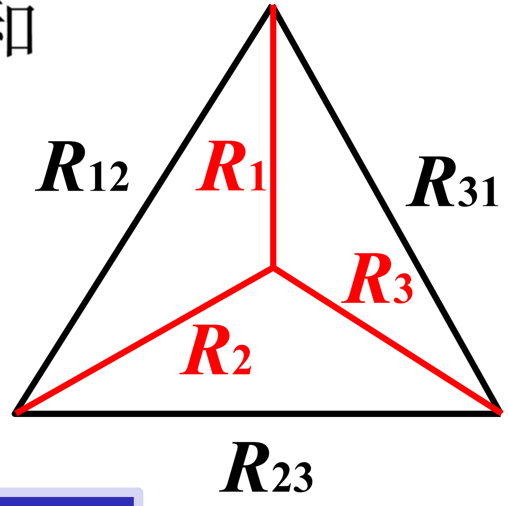
$$R_1 = \frac{R_{12}R_{13}}{R_{12}+R_{13}+R_{23}}$$

$$R_2 = \frac{R_{12}R_{23}}{R_{12}+R_{13}+R_{23}}$$

$$R_3 = \frac{R_{23}R_{13}}{R_{12}+R_{13}+R_{23}}$$

Δ ➔Y：

$$R_i = \frac{\Delta\text{形}i\text{端所联两电阻乘积}}{\Delta\text{形三电阻之和}}$$



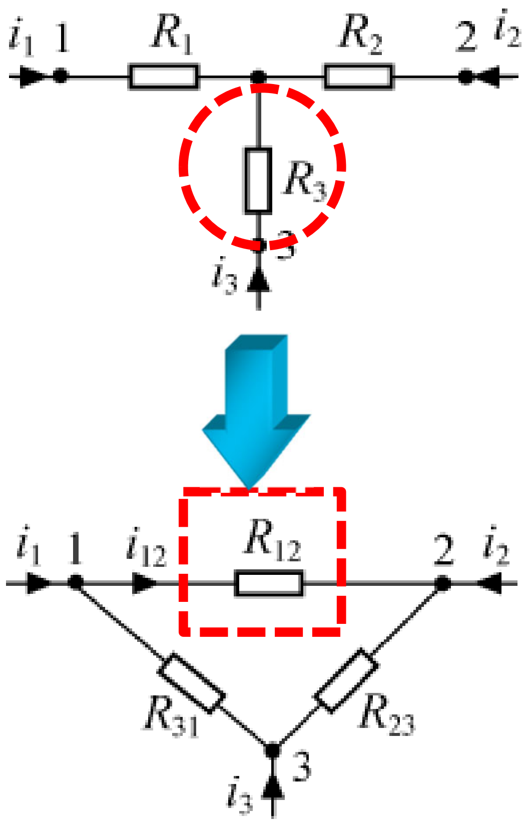
三角形电阻 → 星形电阻连接

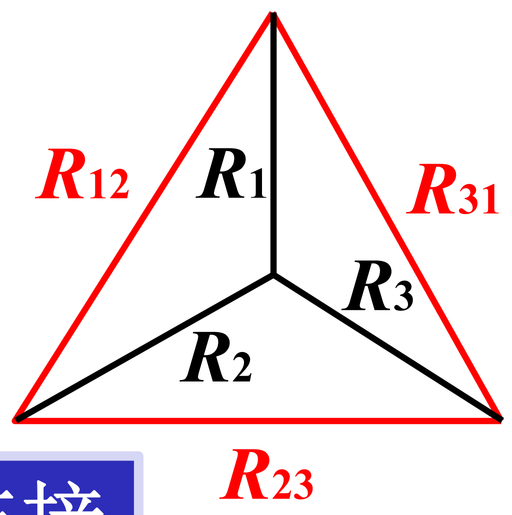
变换公式二

$$R_{12} = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_3}$$

$$R_{23} = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1}$$

$$R_{31} = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_2}$$





星形电阻 → 三角形电阻连接

变换公式二

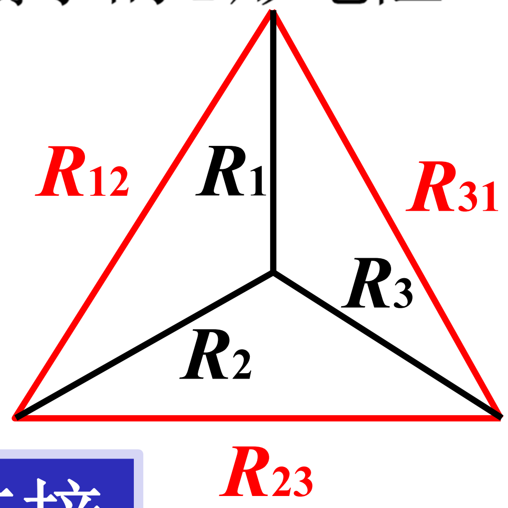
$$R_{12} = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_3}$$

$$R_{23} = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1}$$

$$R_{31} = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_2}$$

Y→Δ：

$$R_{jk} = \frac{\text{Y形电阻两两相乘之和}}{\text{接在与}R_{jk}\text{相对端子的Y形电阻}}$$



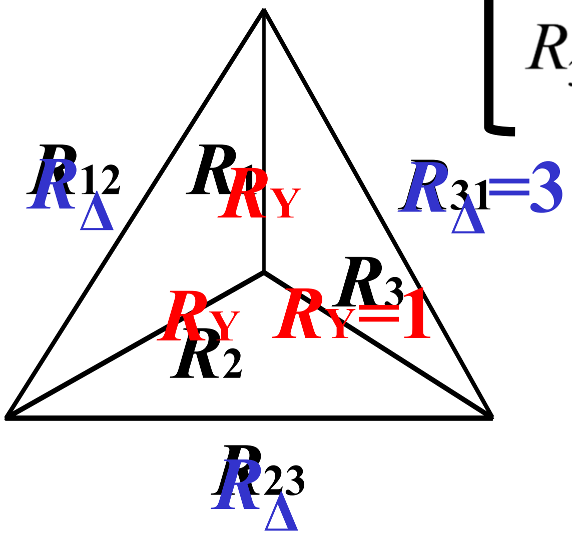
星形电阻 → 三角形电阻连接

特例

如果电阻都相等?

$$\begin{cases} R_{12} = \dfrac{3R_Y^2}{R_Y} \\ R_{23} = \dfrac{R_1R_2 + R_2R_3 + R_1R_3}{R_1} \\ R_{31} = \dfrac{R_1R_2 + R_2R_3 + R_1R_3}{R_2} \end{cases}$$

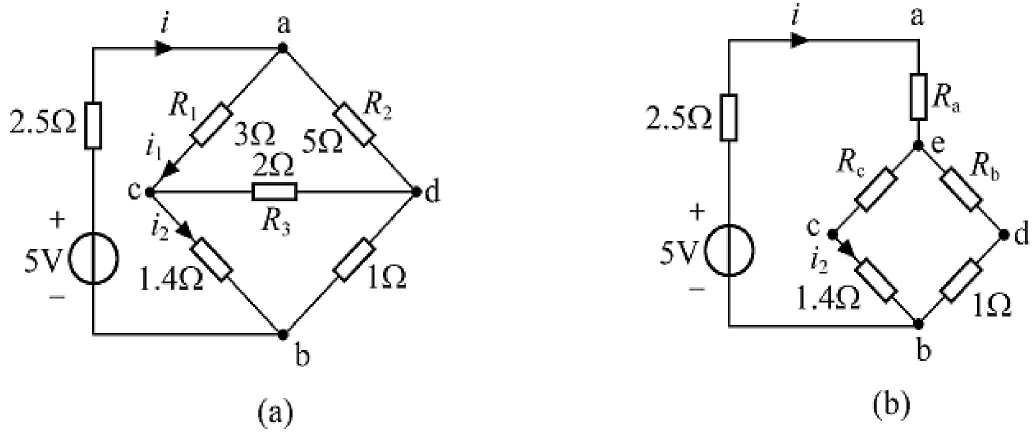
$$\begin{cases} R_1 = \dfrac{R_\Delta^2}{3R_\Delta} \\ R_2 = \dfrac{R_{12}R_{23}}{R_{12} + R_{13} + R_{23}} \\ R_3 = \dfrac{R_{23}R_{13}}{R_{12} + R_{13} + R_{23}} \end{cases}$$



则： $R_\Delta = 3R_Y$

$R_Y = R_\Delta/3$

例 求 i



$$R_a=\frac{R_1R_2}{R_1+R_2+R_3}$$

$$=\frac{3\times5}{3+5+2}=1.5\Omega$$

$$R_b=\frac{2\times5}{3+5+2}=1\Omega \qquad R_c=\frac{2\times3}{10}=0.6\Omega$$

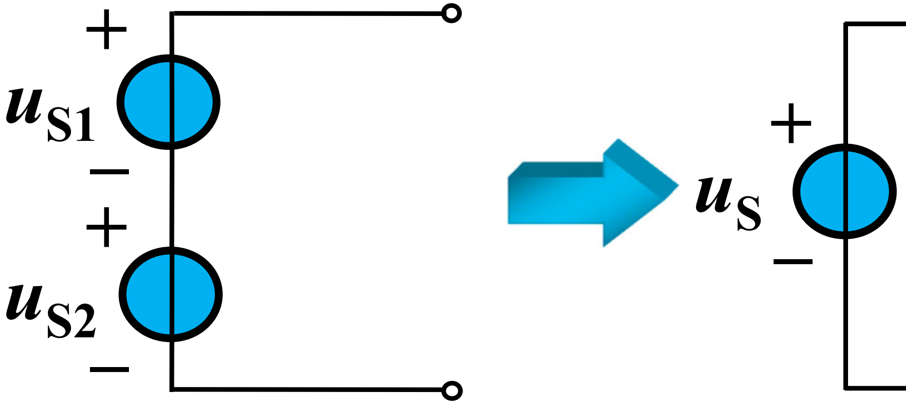
$$R_{ab}=1.5+(0.6+1.4)//(1+1)=2.5\Omega$$

$$I=\frac{5}{R_{ab}+2.5}=1A$$

含独立源网络的等效变换

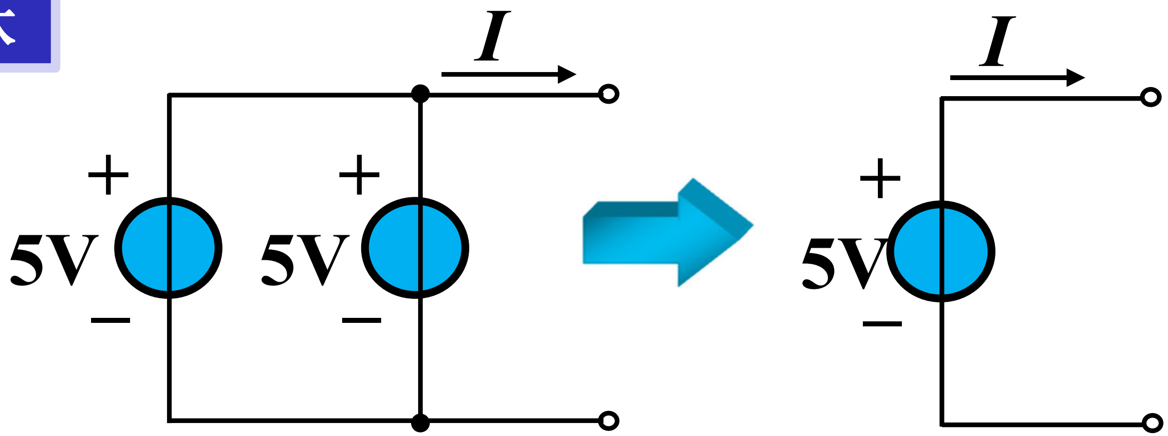
电压源的串并联

串联



$$u_S = u_{S1} + u_{S2}$$

并联

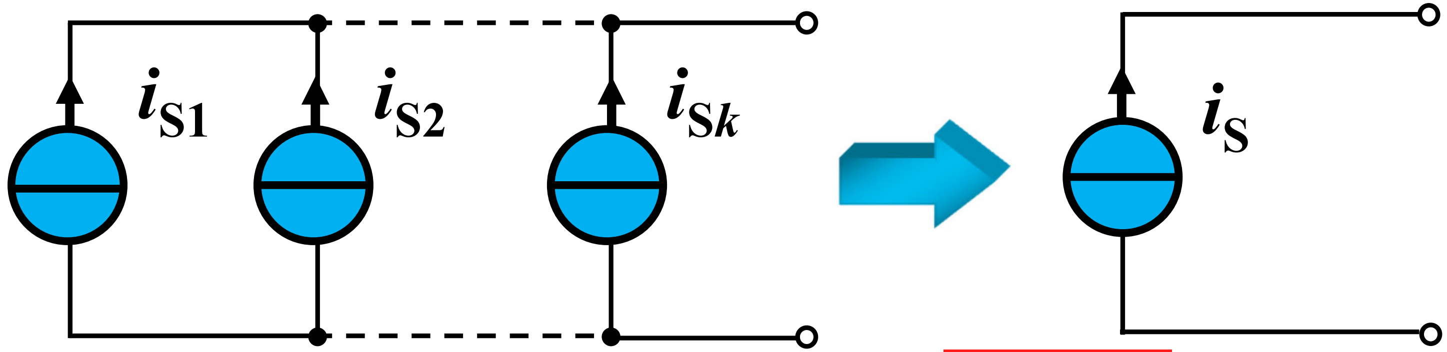


注意

电压相同的电压源才能并联，且每个电源的电流不确定。

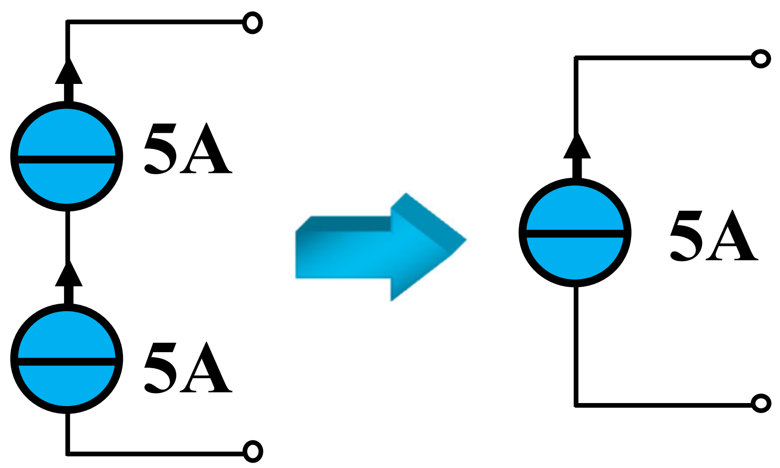
电流源的串并联

并联



$$i_S = \sum i_{S_k}$$

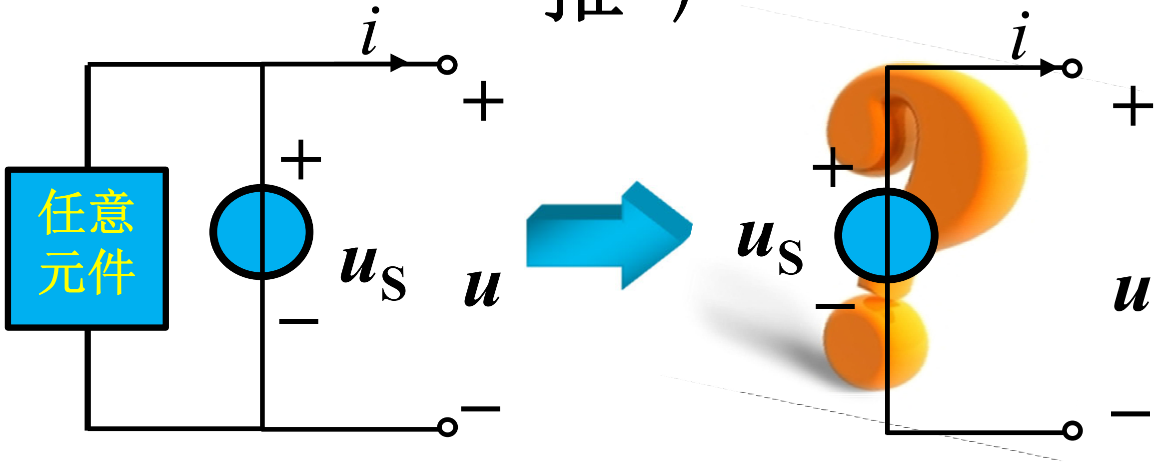
串联

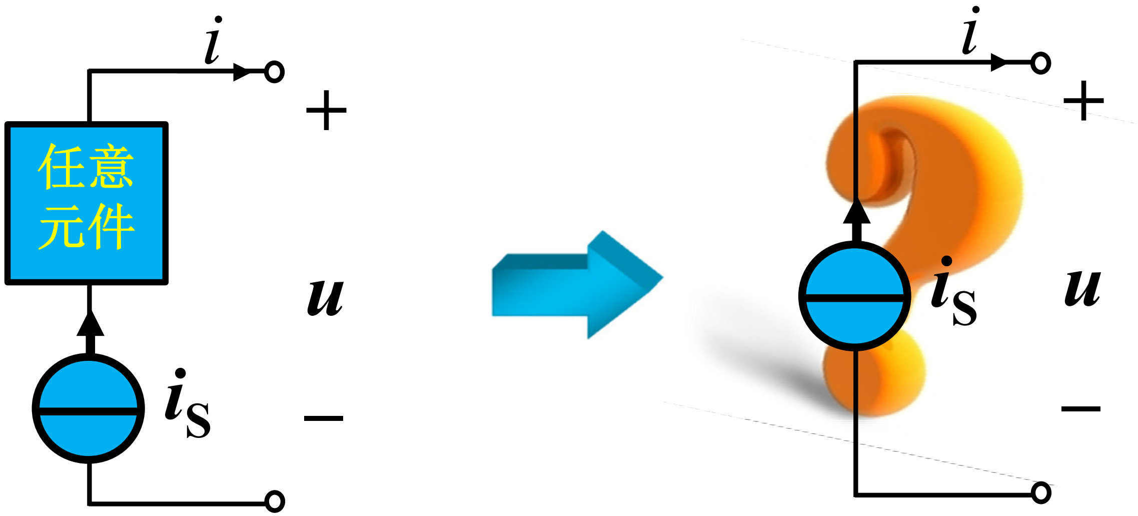


注意

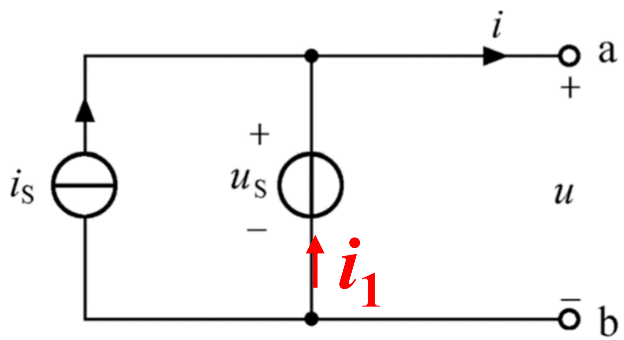
电流相同的电流源才能串联，且每个电源上的电压不确定。

推 广

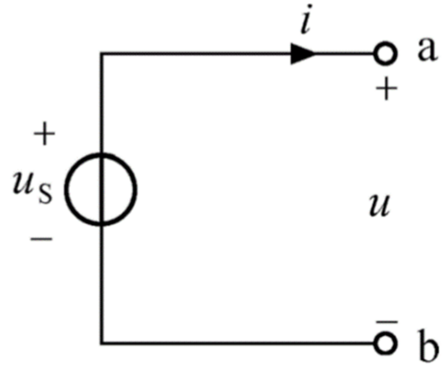




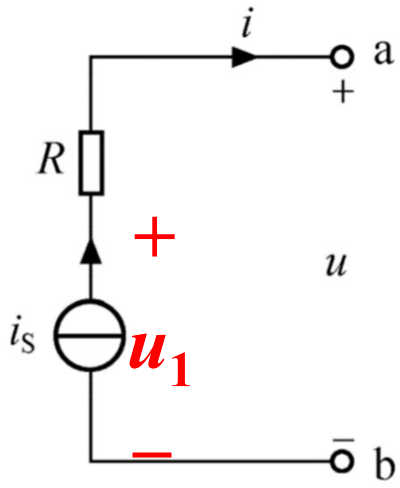
◆含独立源网络的等效变换



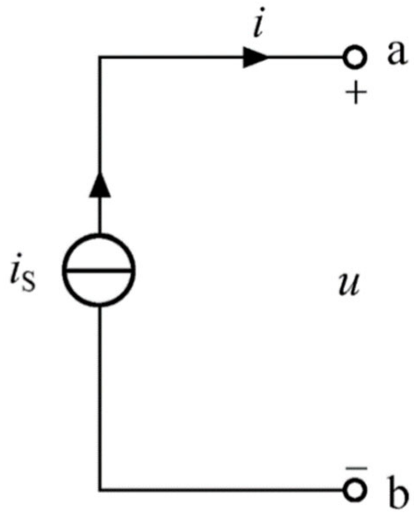
(a)



(b)



(a)



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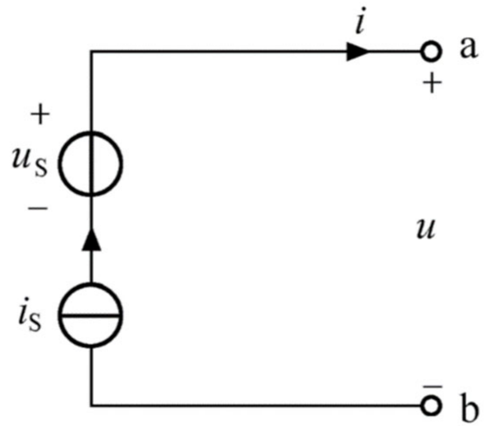
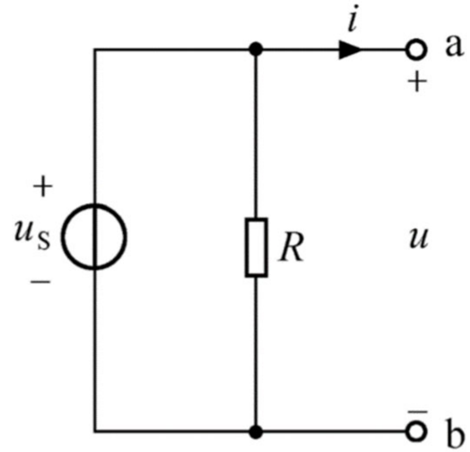


(a)



(a)

注意

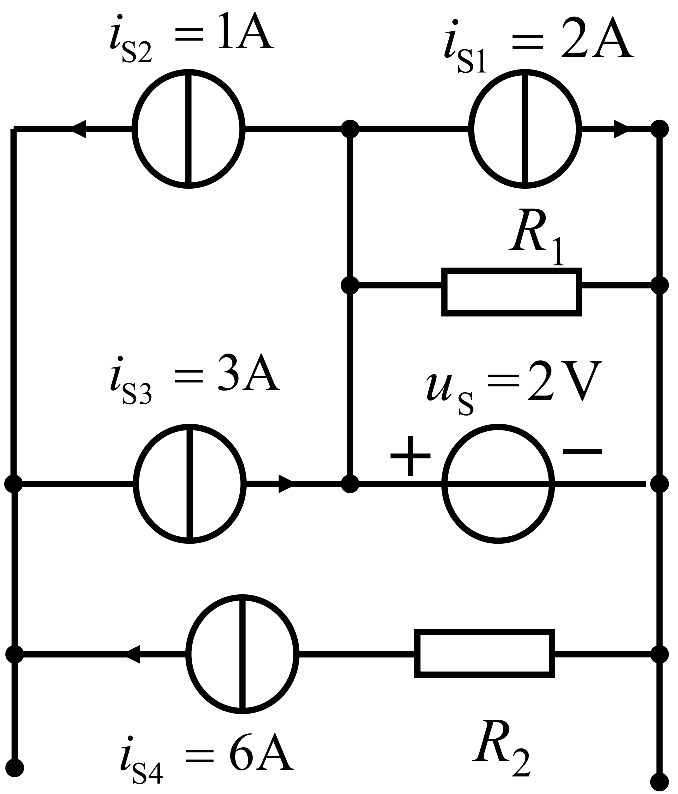
➢ 对外等效

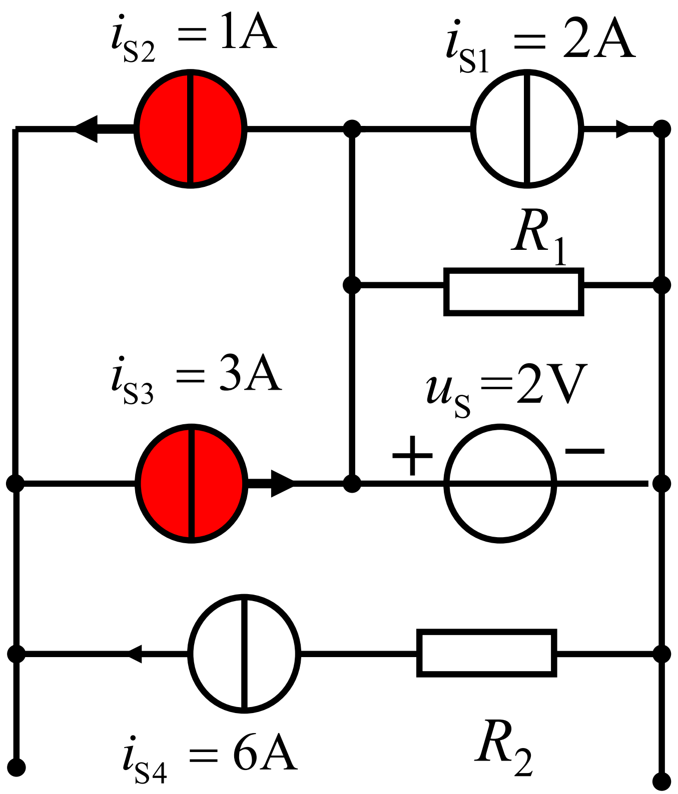
电路的等效变换只改变电路的内部结构，但保持其端口上的电压和电流的关系（*VCR*）不变；

➢ 对内不等效

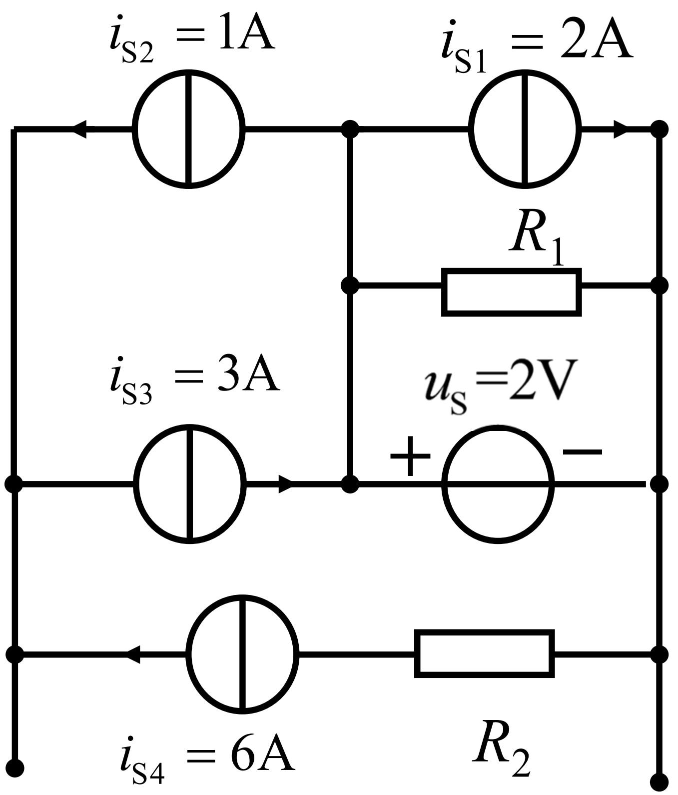
对被变换的电路部分而言，与原电路的工作状态不同。

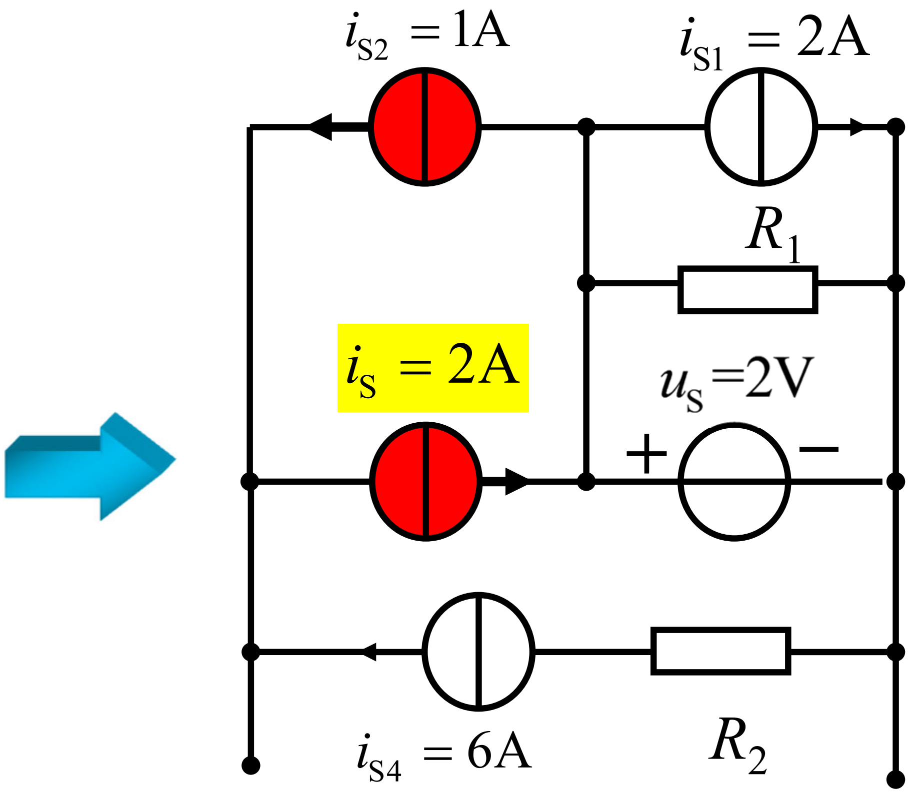
【例】（P35例2-7）将下图等效简化为一个电压源或者电流源。



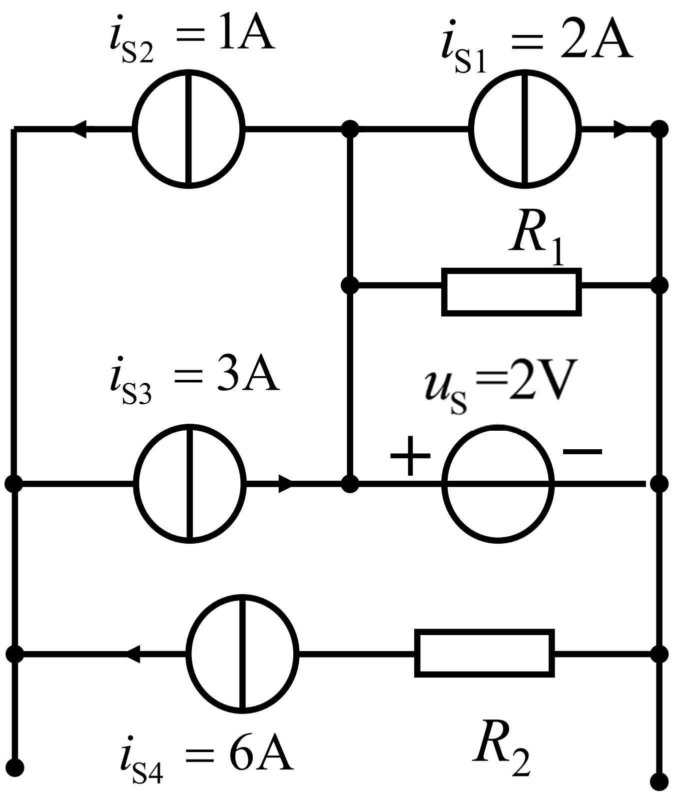


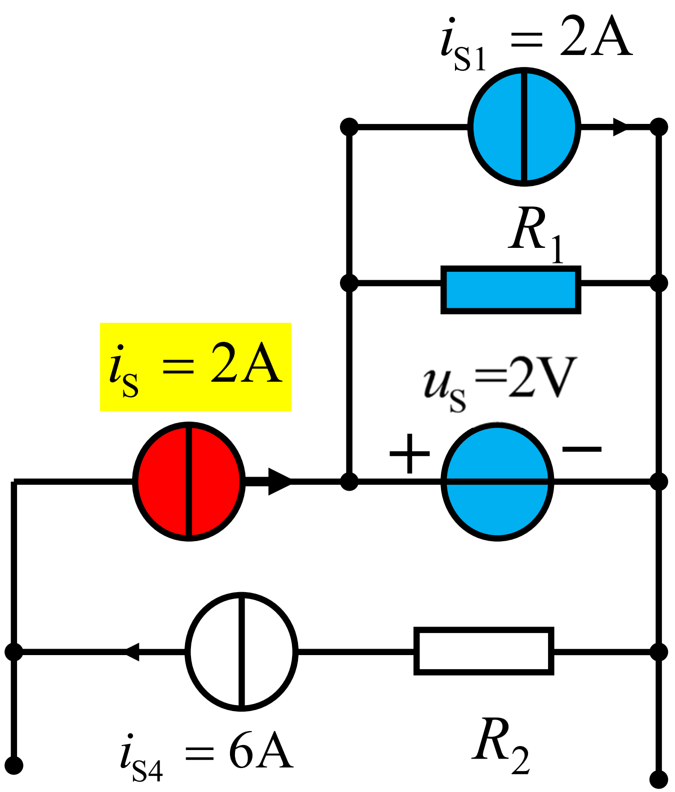
【例】（P35例2-7）将下图等效简化为一个电压源或者电流源。



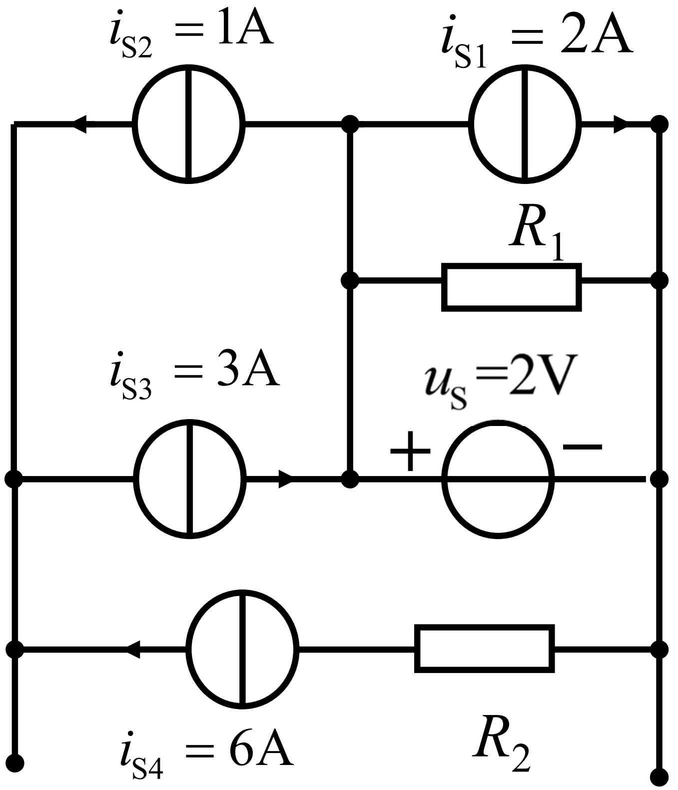


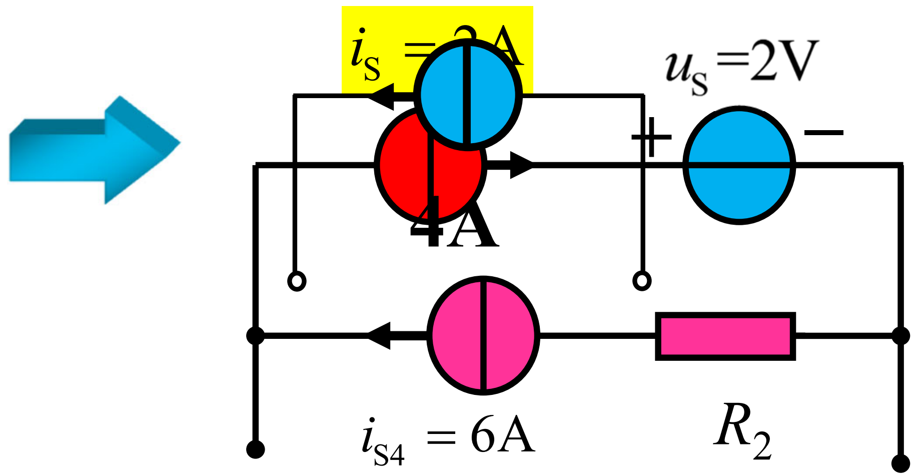
【例】（P35例2-7）将下图等效简化为一个电压源或者电流源。





【例】（P35例2-7）将下图等效简化为一个电压源或者电流源。



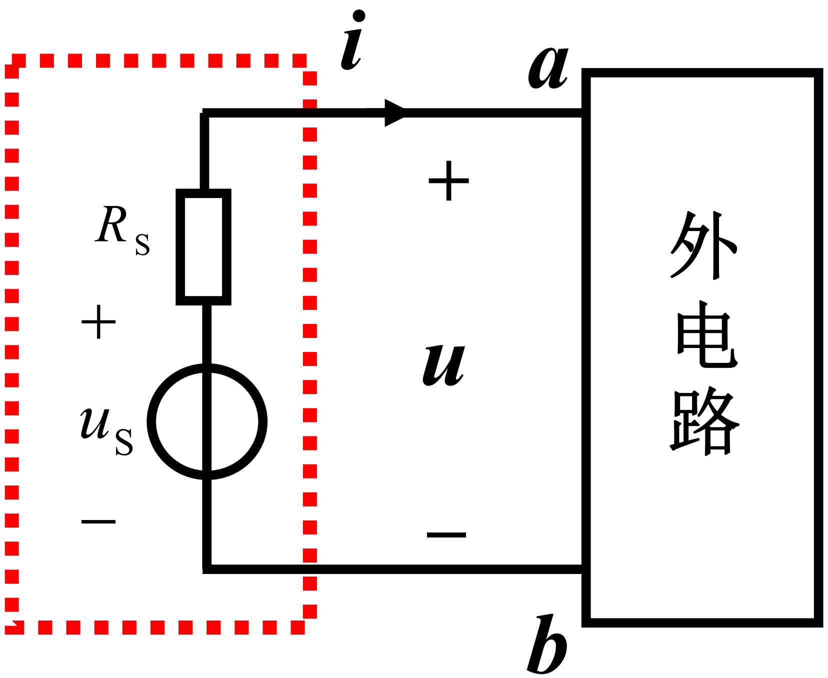


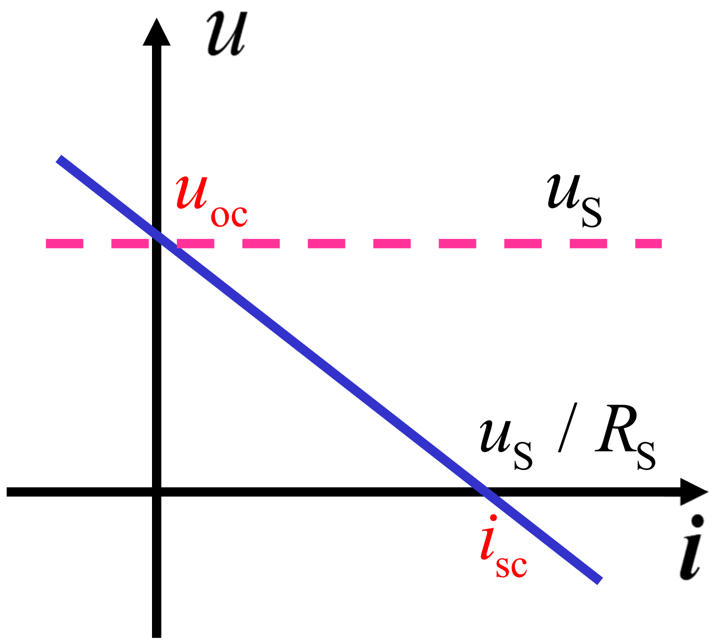
总结：

1.电压源串联等效为各个电压源的代数和。
2.电流源并联等效为各个电流源的代数和。
3.电压源与任何二端网络并联等效为此电压源。
4.电流源与任何二端网络串联等效为此电流源。

实际电源的两种模型及其等效变换

实际电压源模型(戴维南模型)

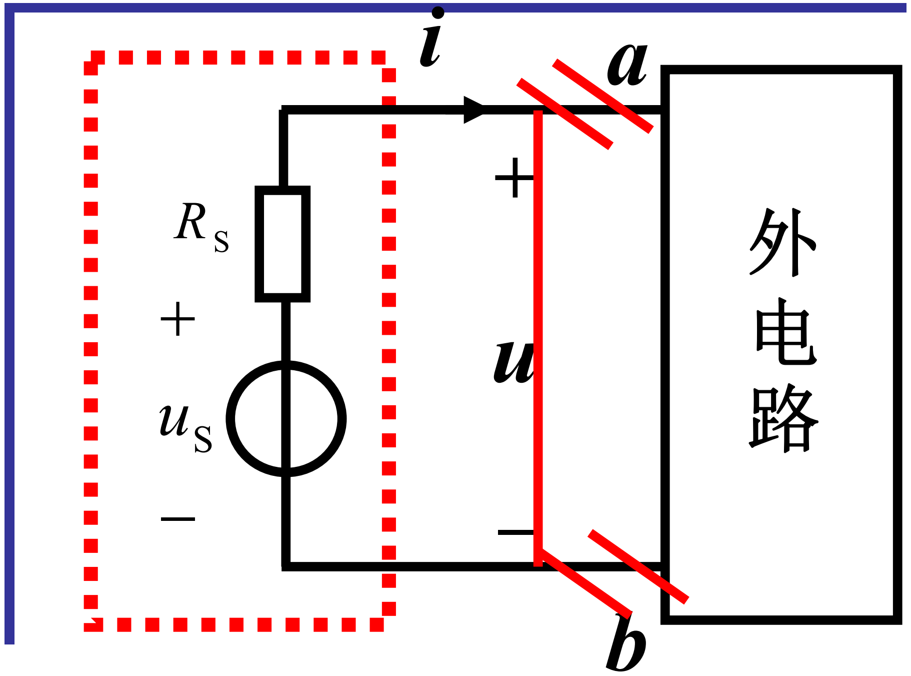


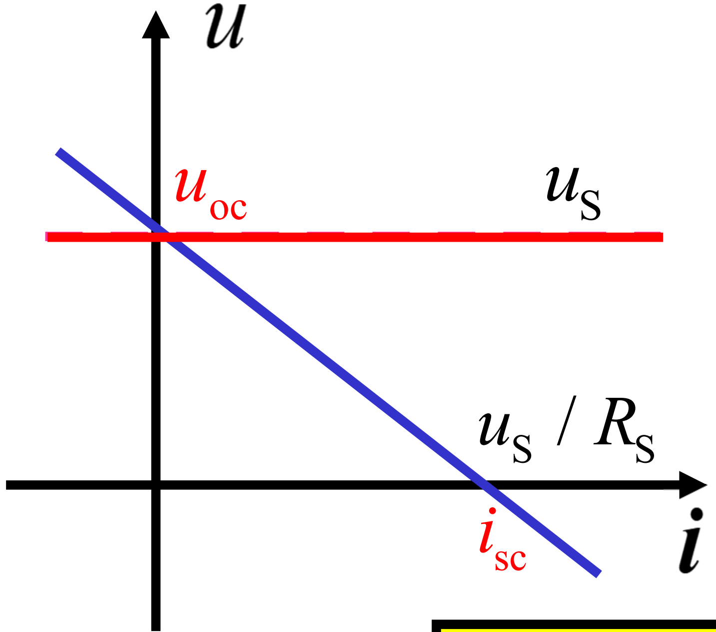


VCR

$$u = u_S - R_S i$$

实际电压源模型电路特性

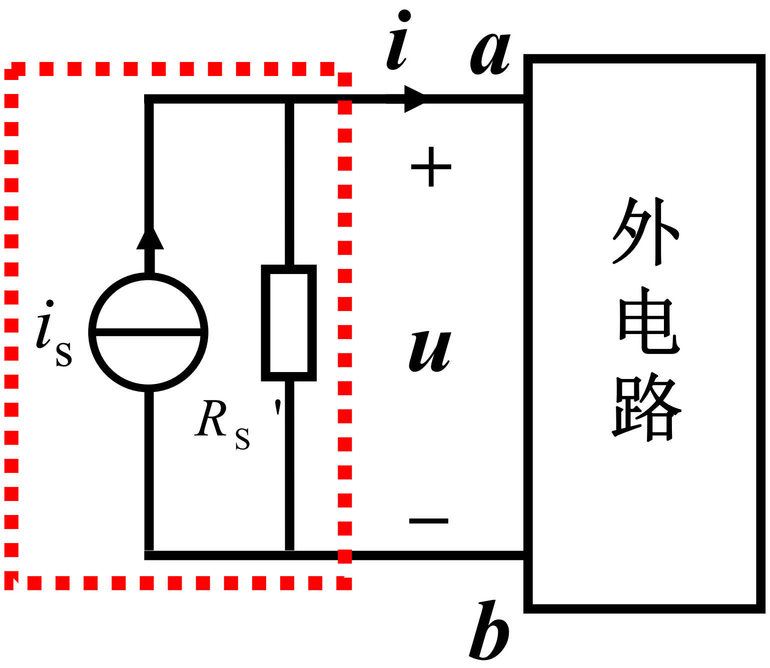


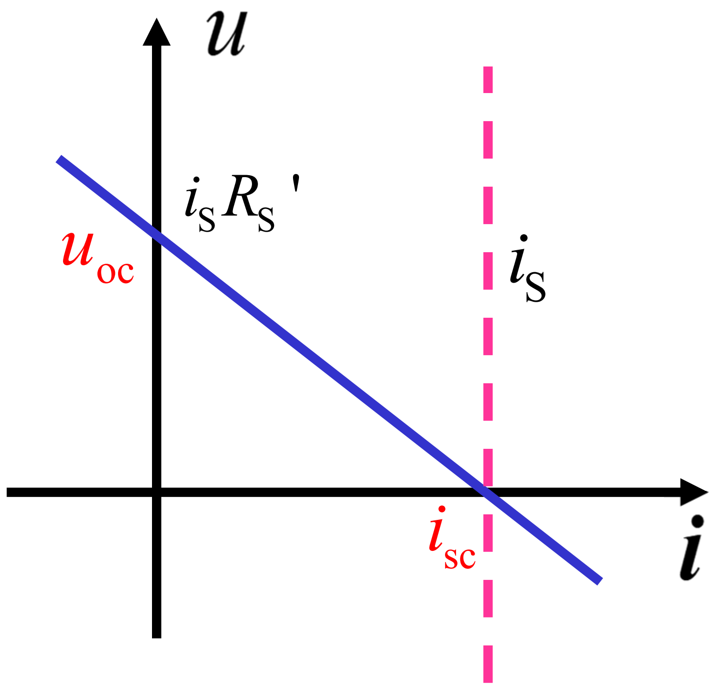


$$u = u_S - R_S i$$

- i 增大，R_s压降增大，u 减小；
- $i = 0$，$u = u_s = u_{oc}$，开路电压；
- $u = 0$，$i = i_{sc} = u_s$ /R_s，短路电流，实际情况中不允许电压源短路；
- $R_s = 0$，理想电压源。

实际电流源模型(诺顿模型)

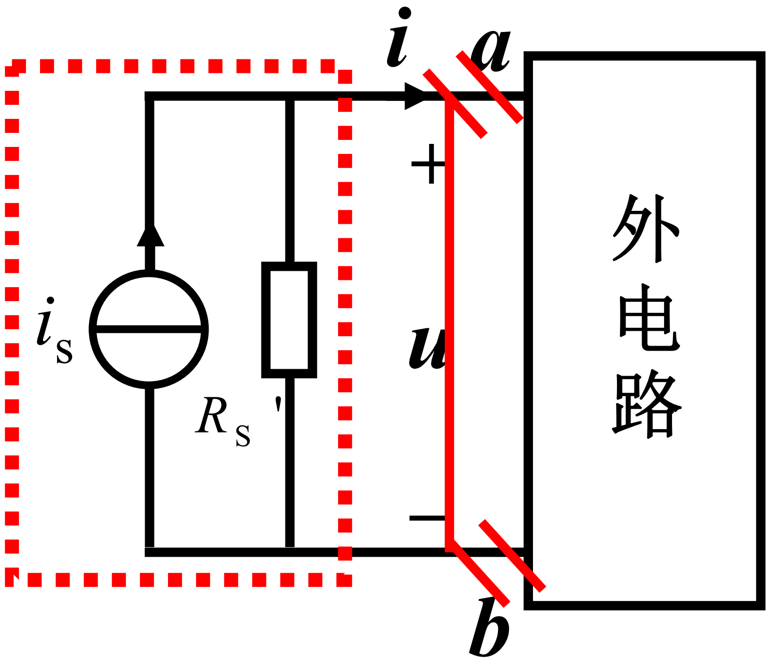


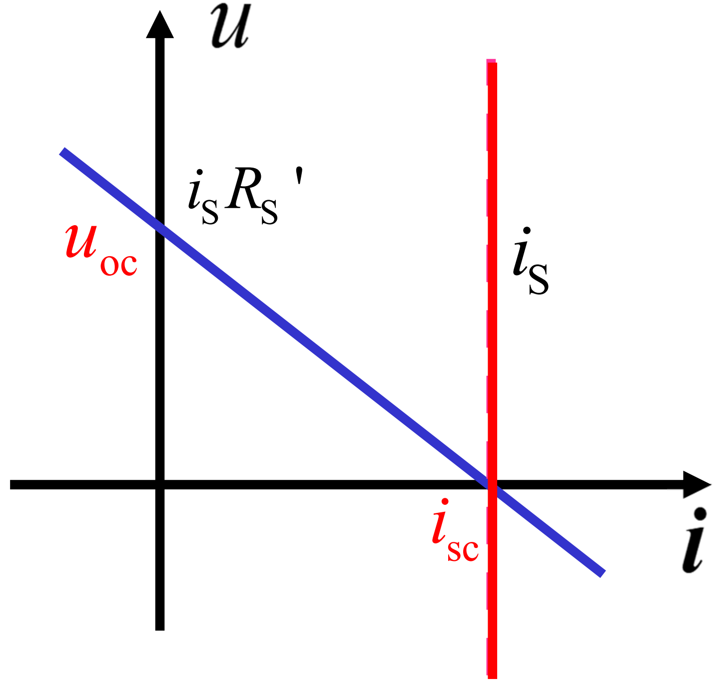


VCR

$$i = i_S - u / R_S'$$

实际电流源模型电路特性

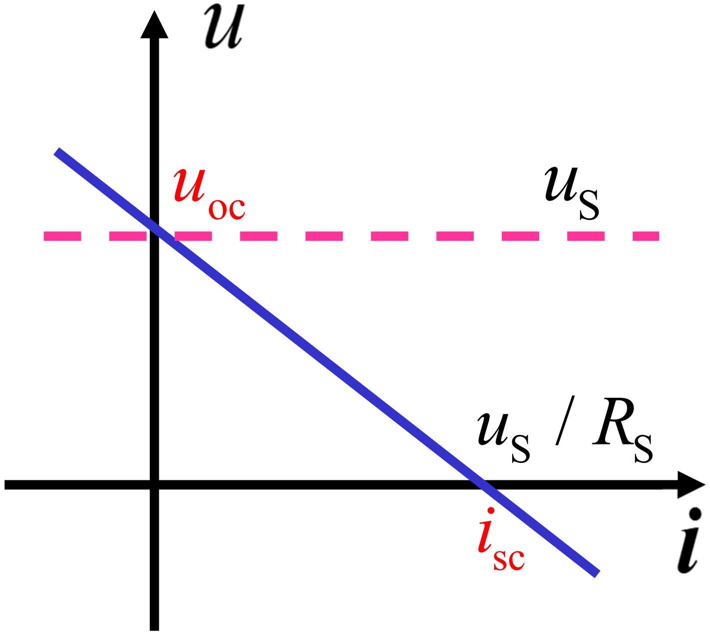


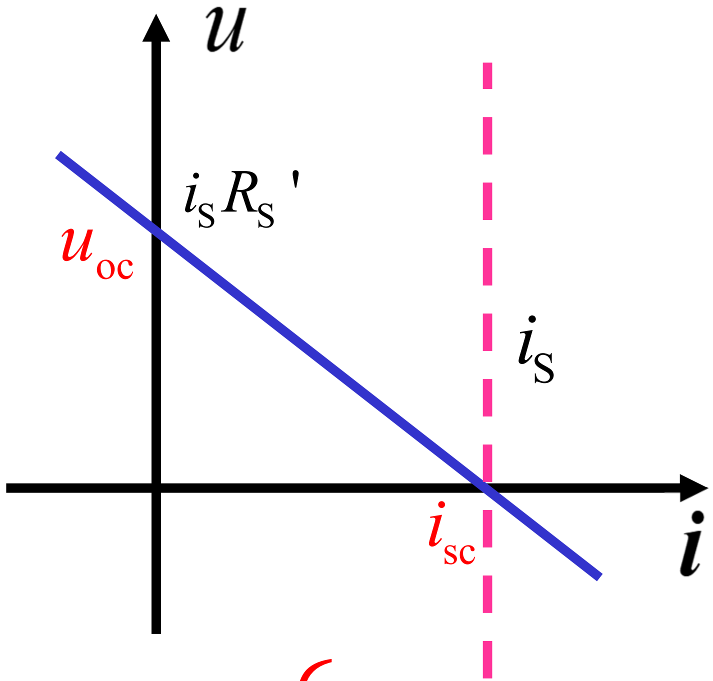


- ➢ u 增大，R_S '分流增大，i 减小
- ➢ $i=0$，$u=u_{oc}=i_S R_S'$ 开路电压
- ➢ $u=0$，$i=i_{sc}=i_S$，短路电流
- ➢R_S '为‘无穷大’，理想电流源

$$i=i_S-u/R_S'$$

两种实际电源模型的等效变换



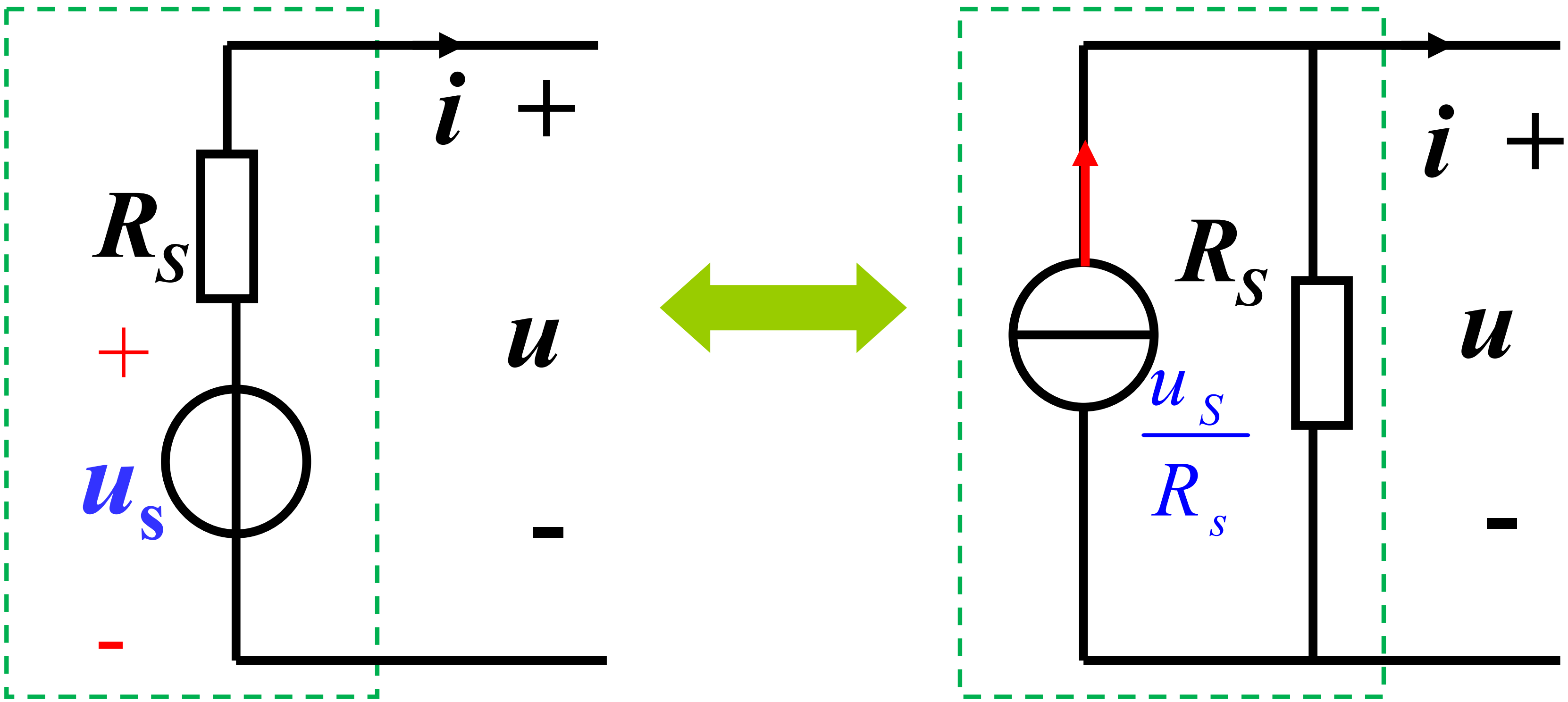


$$\begin{cases} u = u_s - R_s i \\ u = R'_s i_s - R'_s i \end{cases}$$

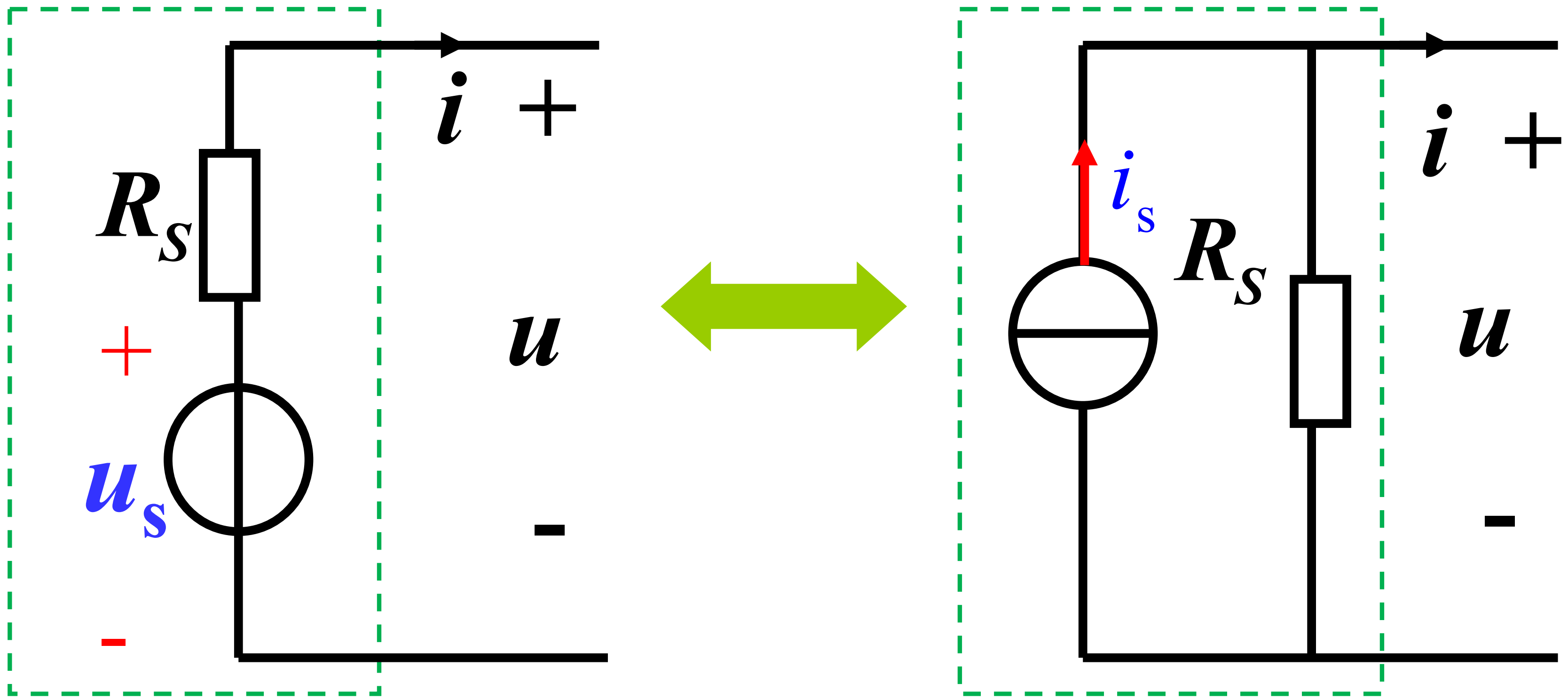
等效转换条件

$$\begin{cases} i_s = u_s / R_S \\ R_S = R_S' \end{cases}$$

◆实际电源的两种模型及其等效变换



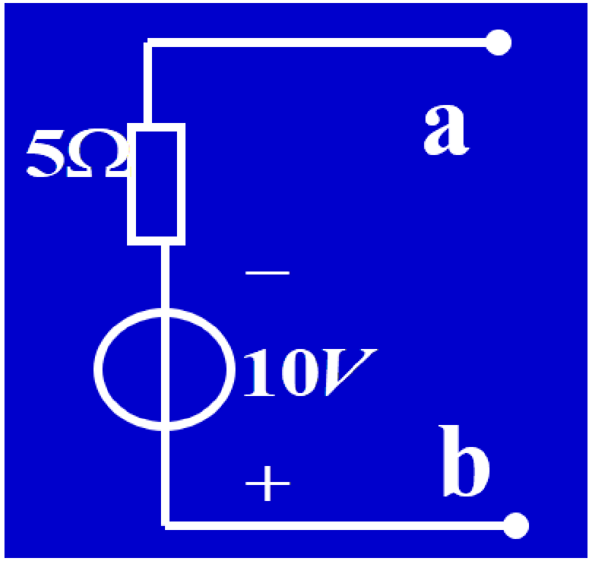
$$
\begin{cases}
i_s = u_s / R_S \\
R_S = R_S'
\end{cases}
$$

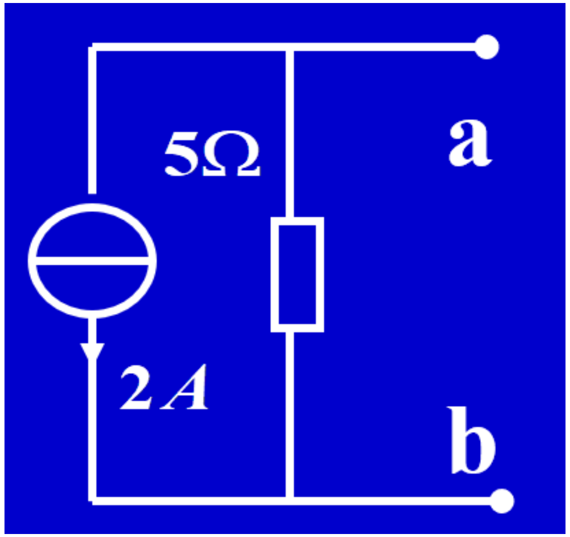


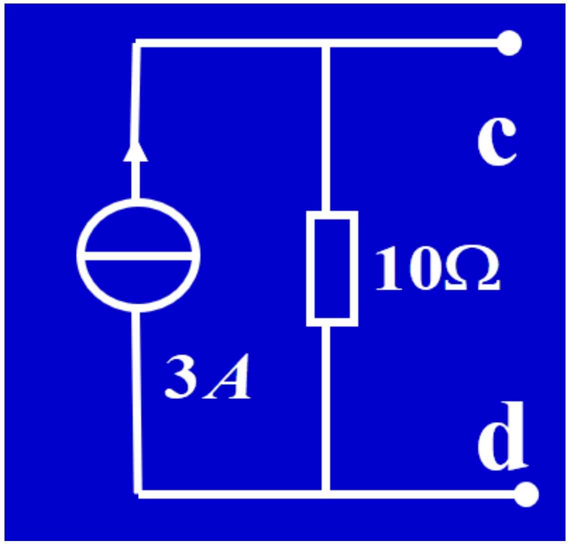
（1）对外等效，对内不等效

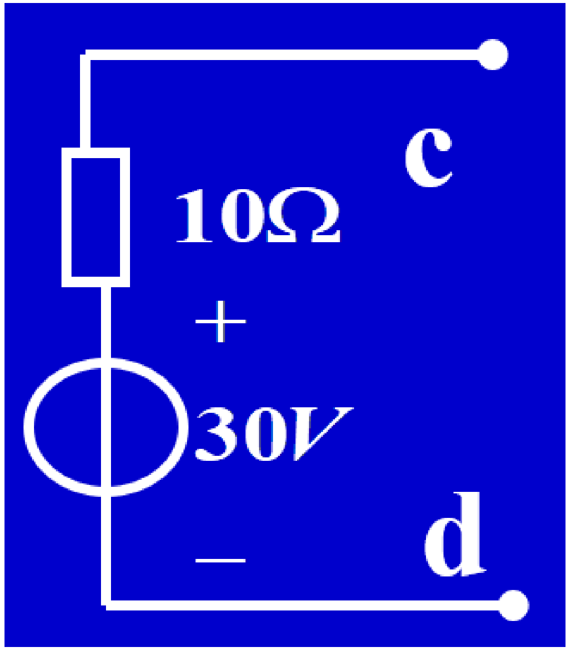
（2）理想电压源，R_s =0， 两种电源模型不能等效转换。

$$\begin{cases} i_s = u_s / R_S \\ R_S = R_S' \end{cases}$$

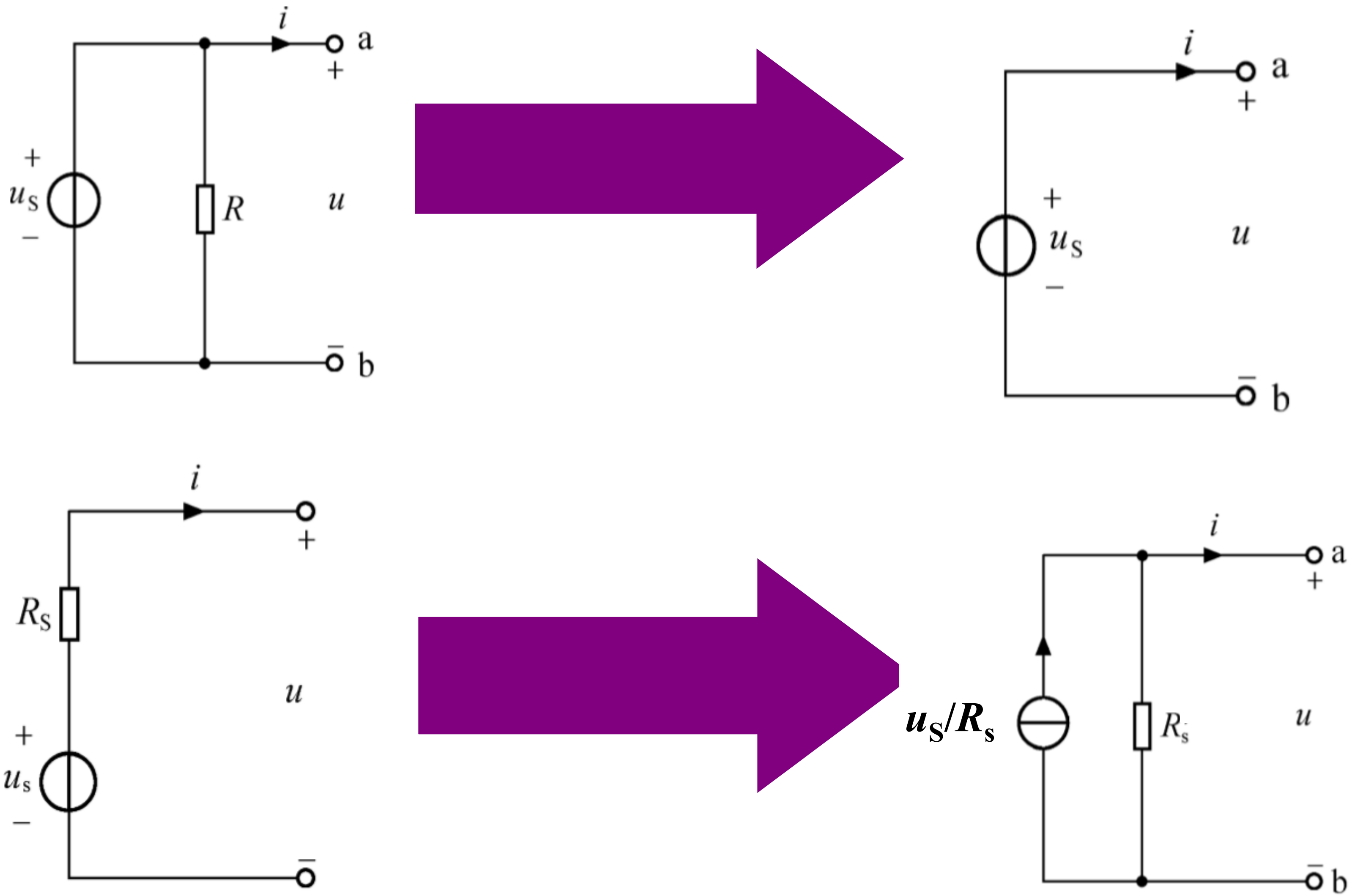

5Ω
a
—
10V
+
b


5Ω
a
2A
b

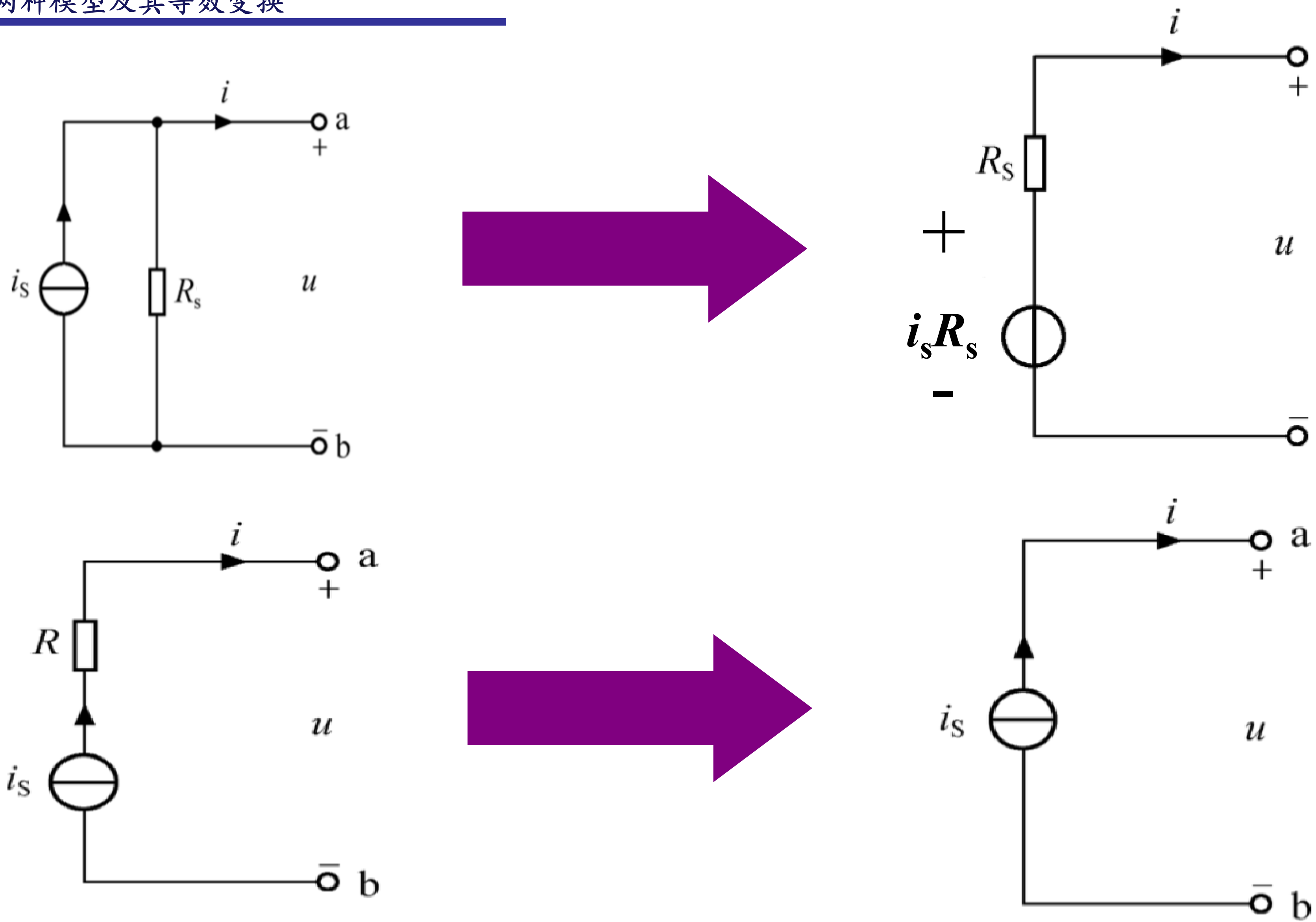

c
10Ω
3A
d


c
10Ω
+
30V
—
d

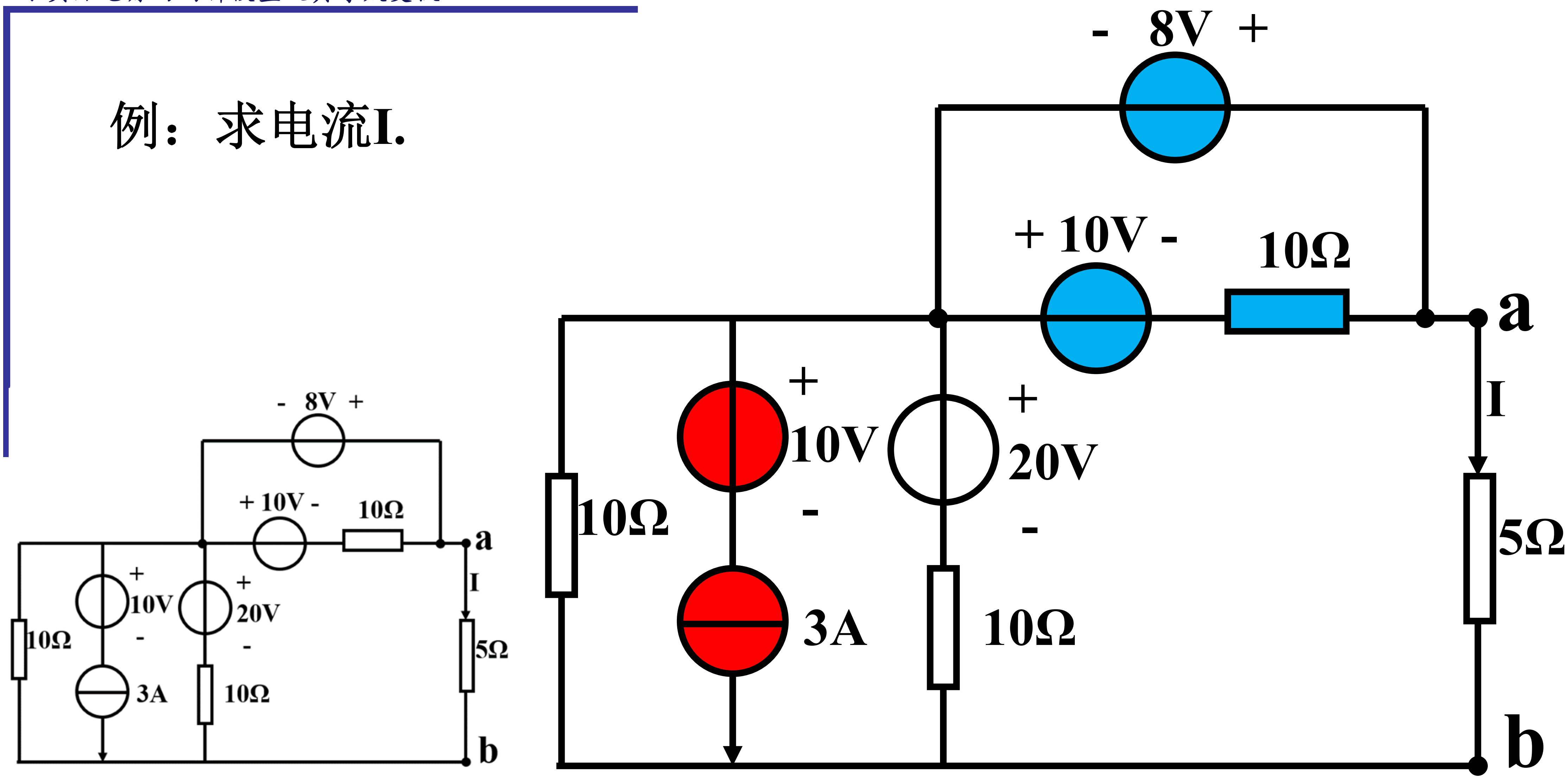
◆实际电源的两种模型及其等效变换



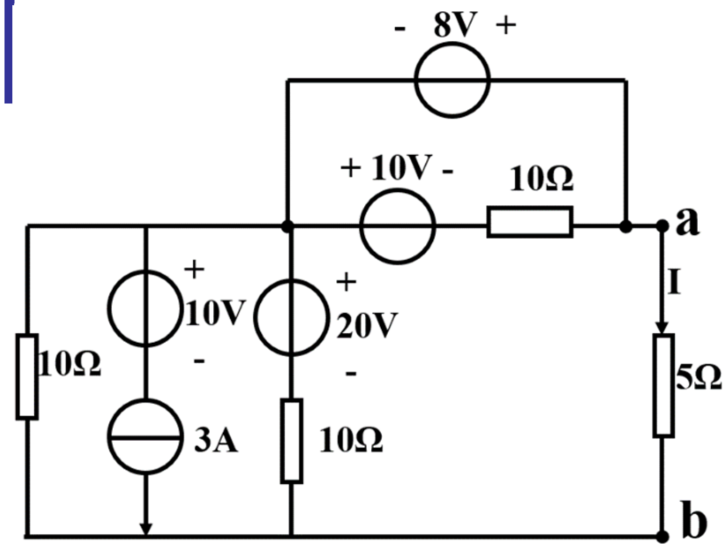
◆实际电源的两种模型及其等效变换

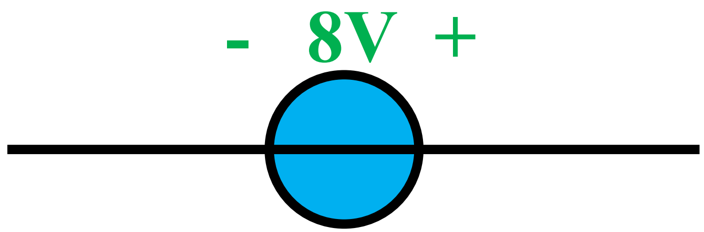


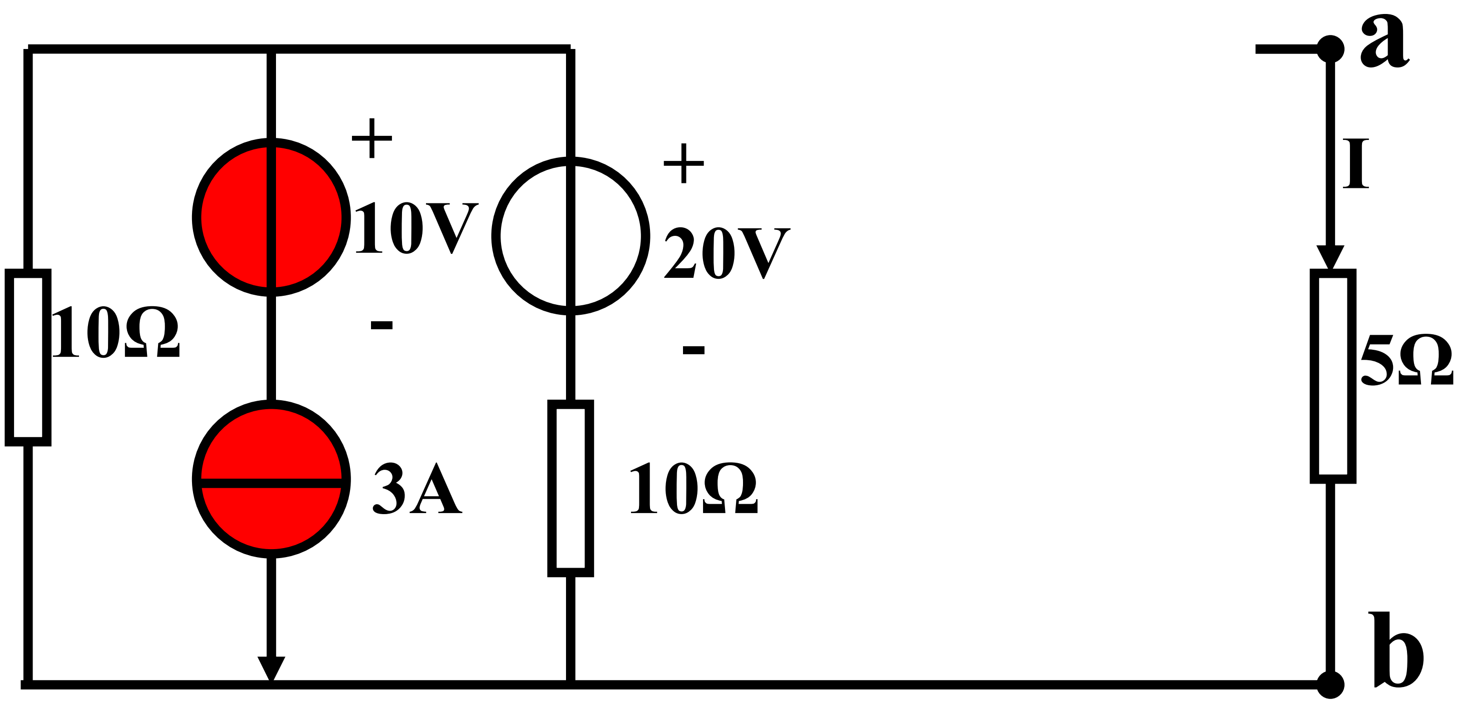
例：求电流I.


- 8V +
+ 10V -
10Ω
a
I
+
10V
-
+
20V
-
10Ω
3A
10Ω
5Ω
b
- 8V +
+ 10V -
10Ω
a
+
10V
-
+
20V
-
I
10Ω
5Ω
3A
10Ω
b

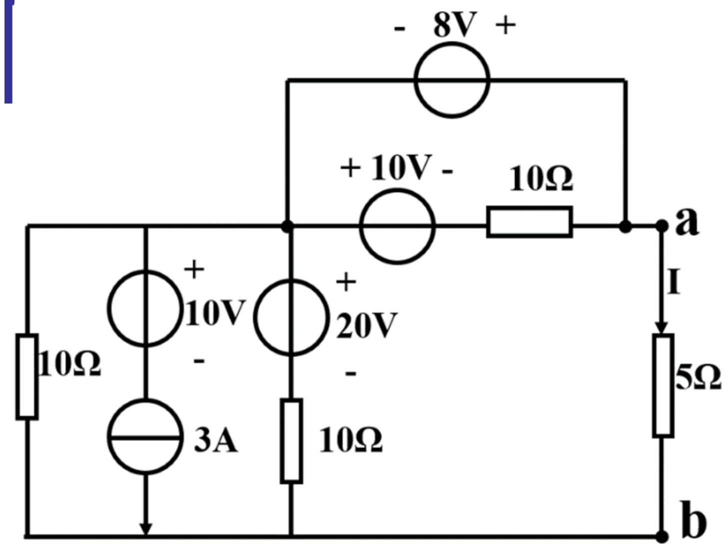
例：求电流I.

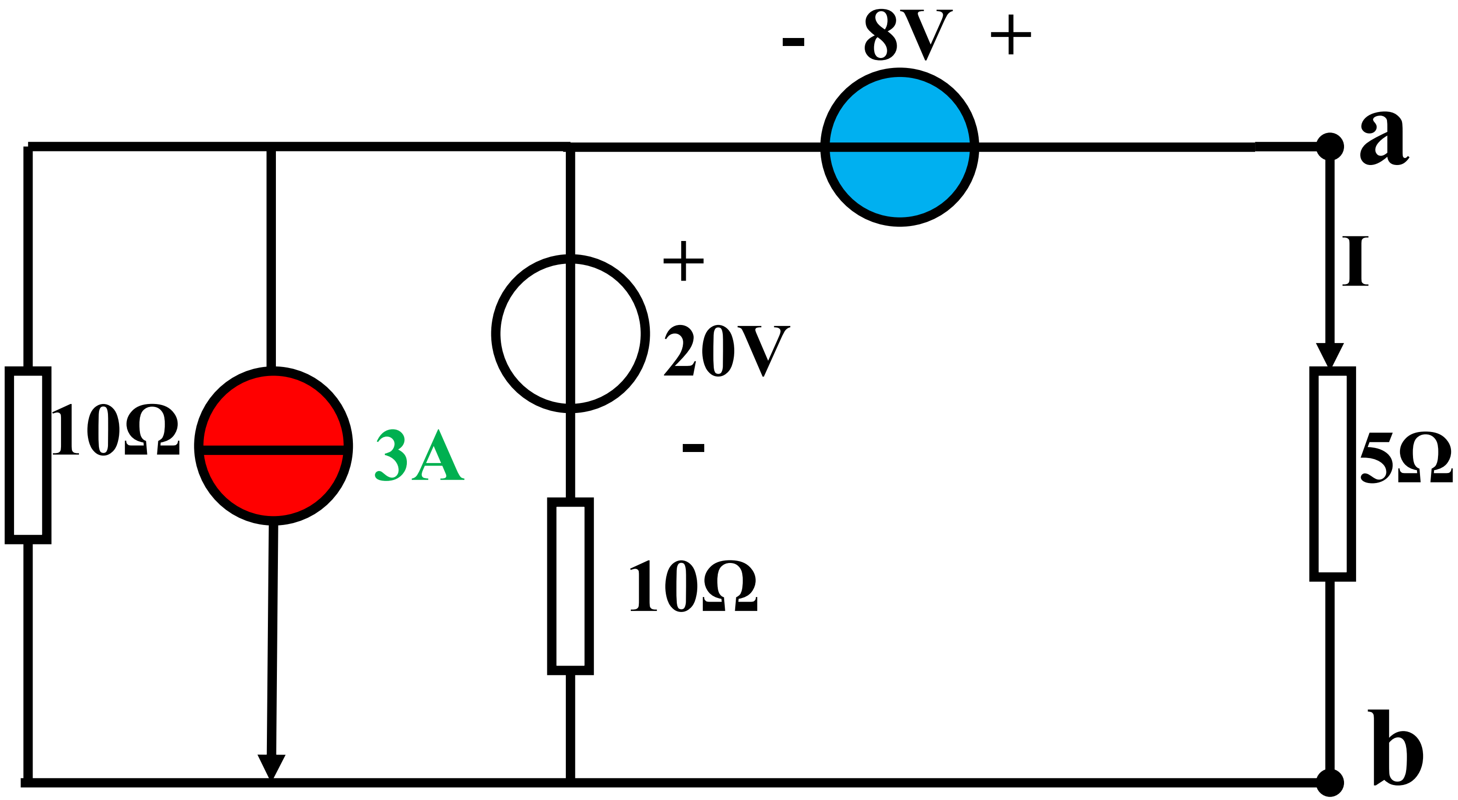




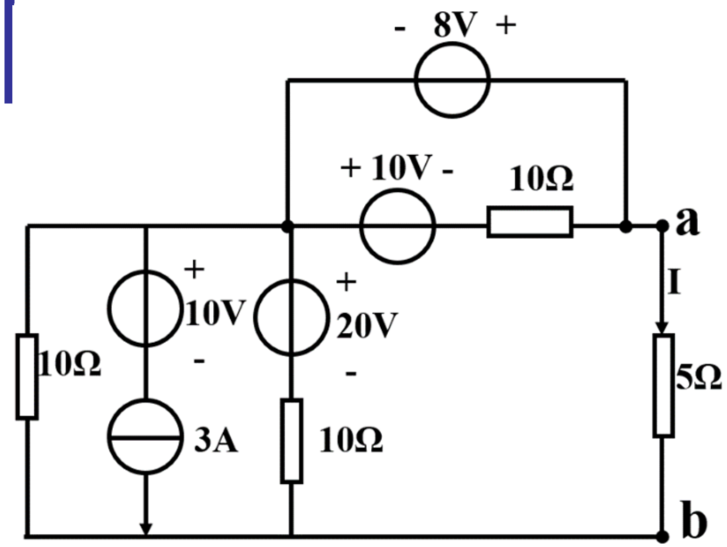


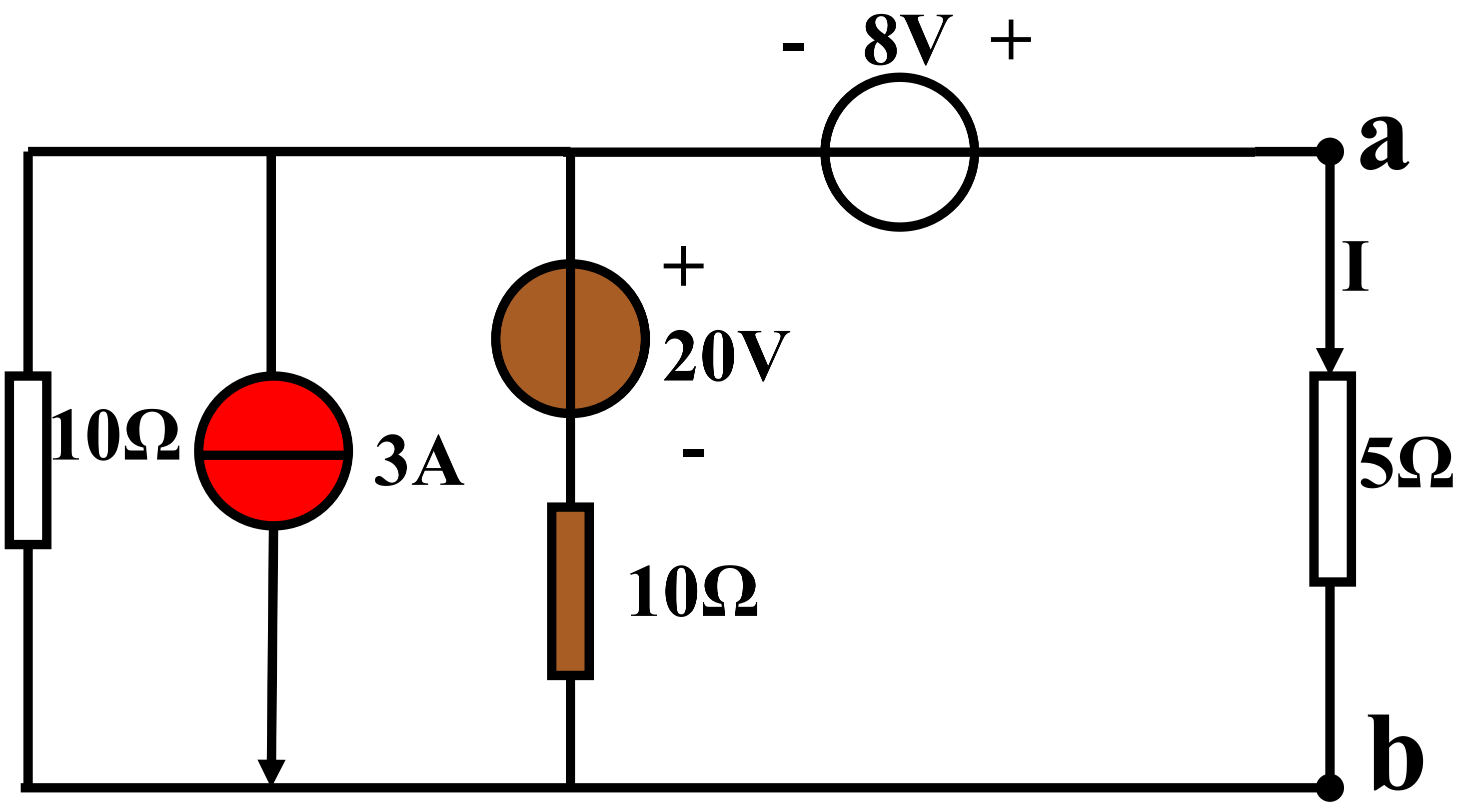
例：求电流I.



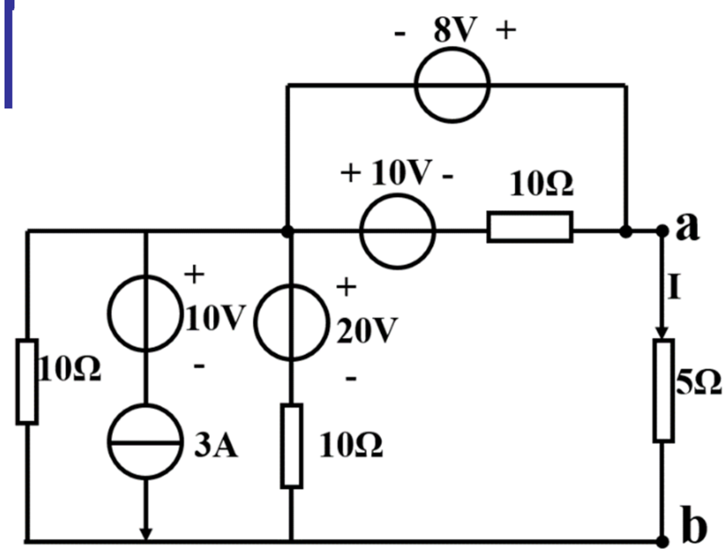


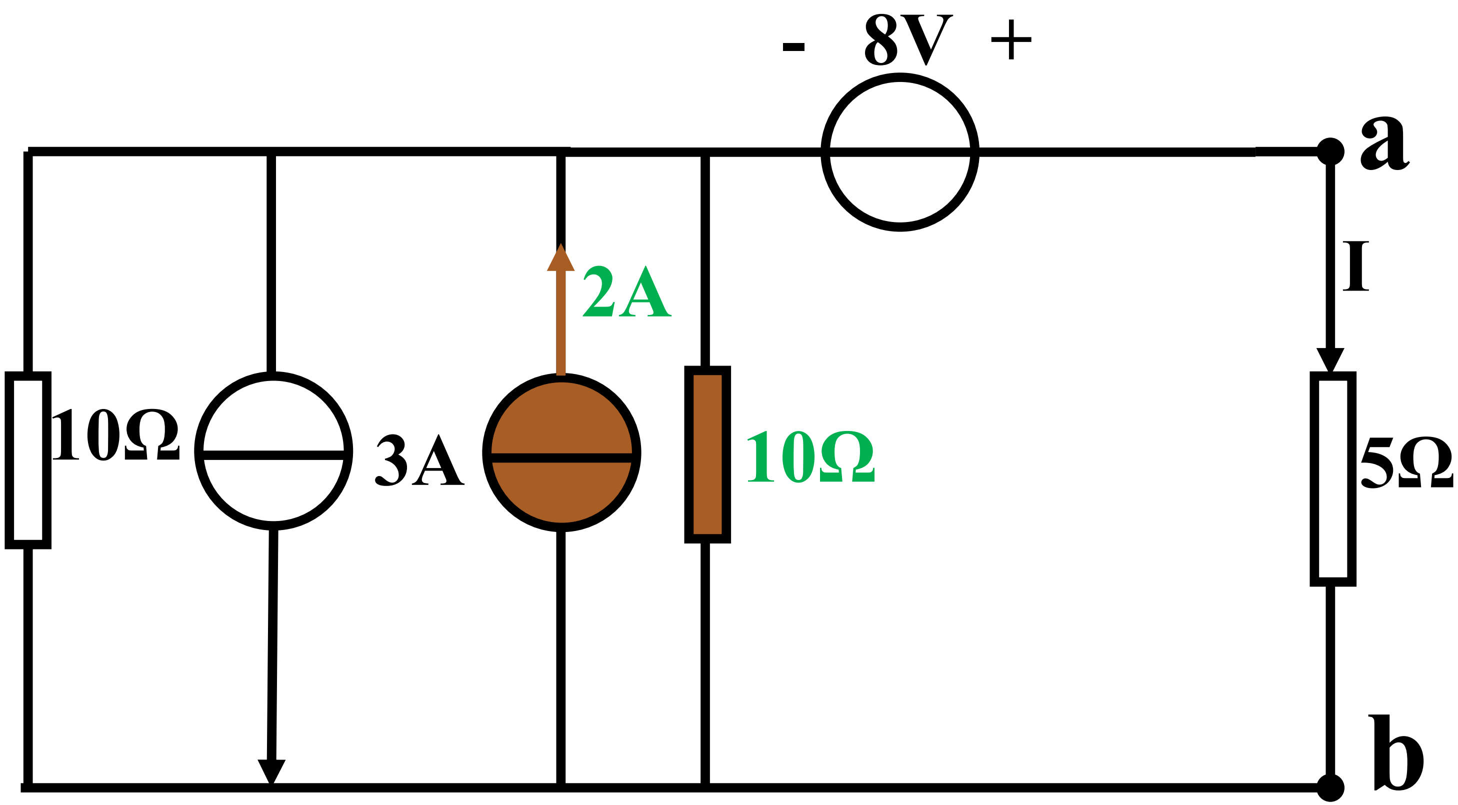
例：求电流I.



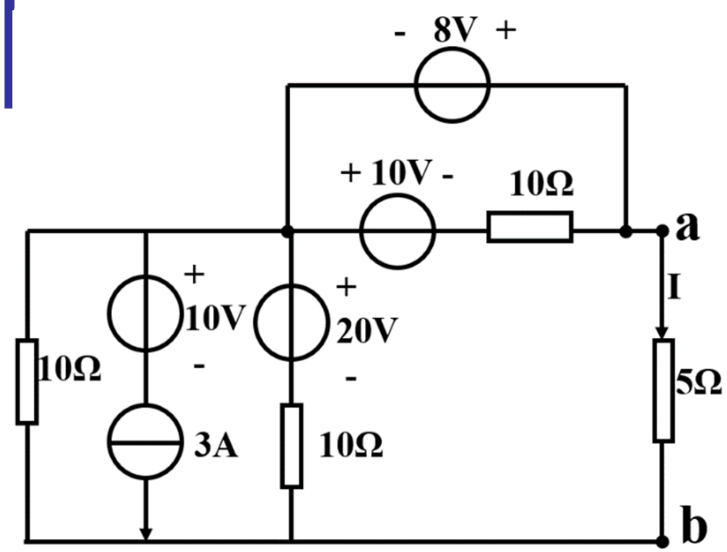


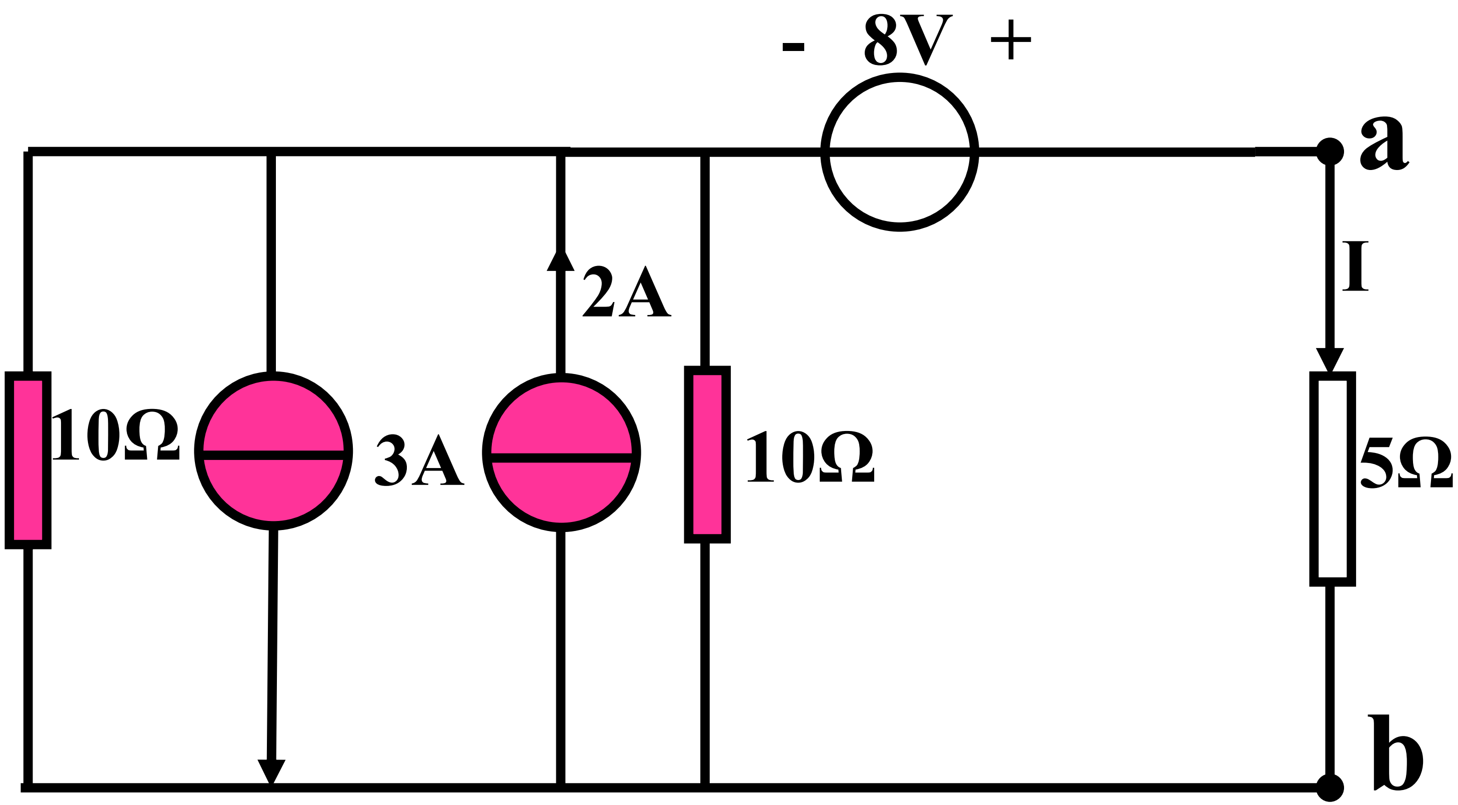
例：求电流I.



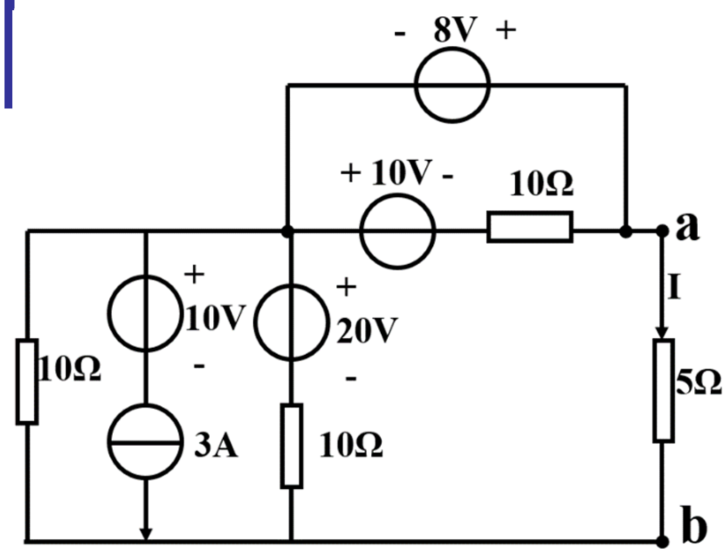


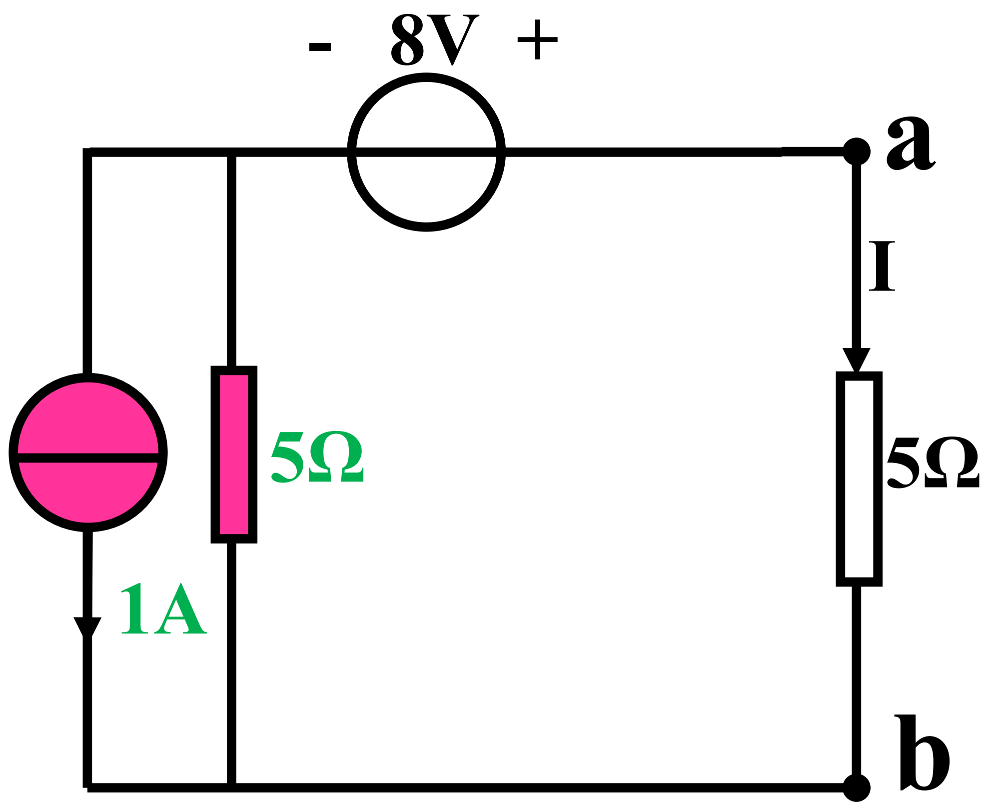
例：求电流I.





例：求电流I.

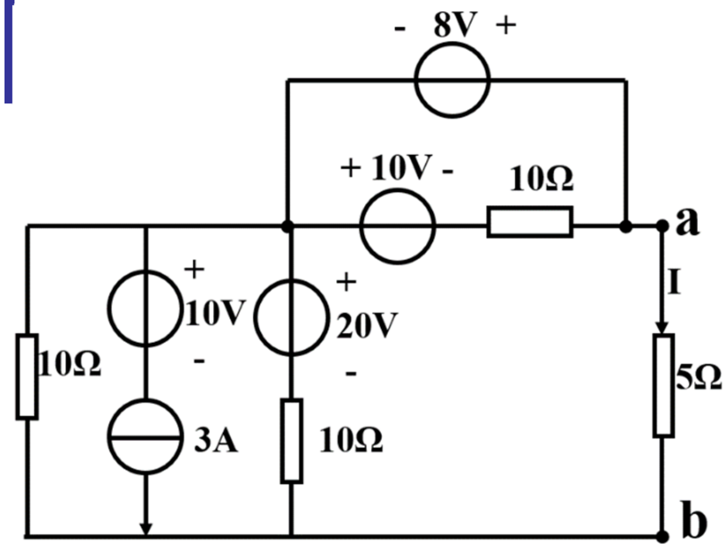


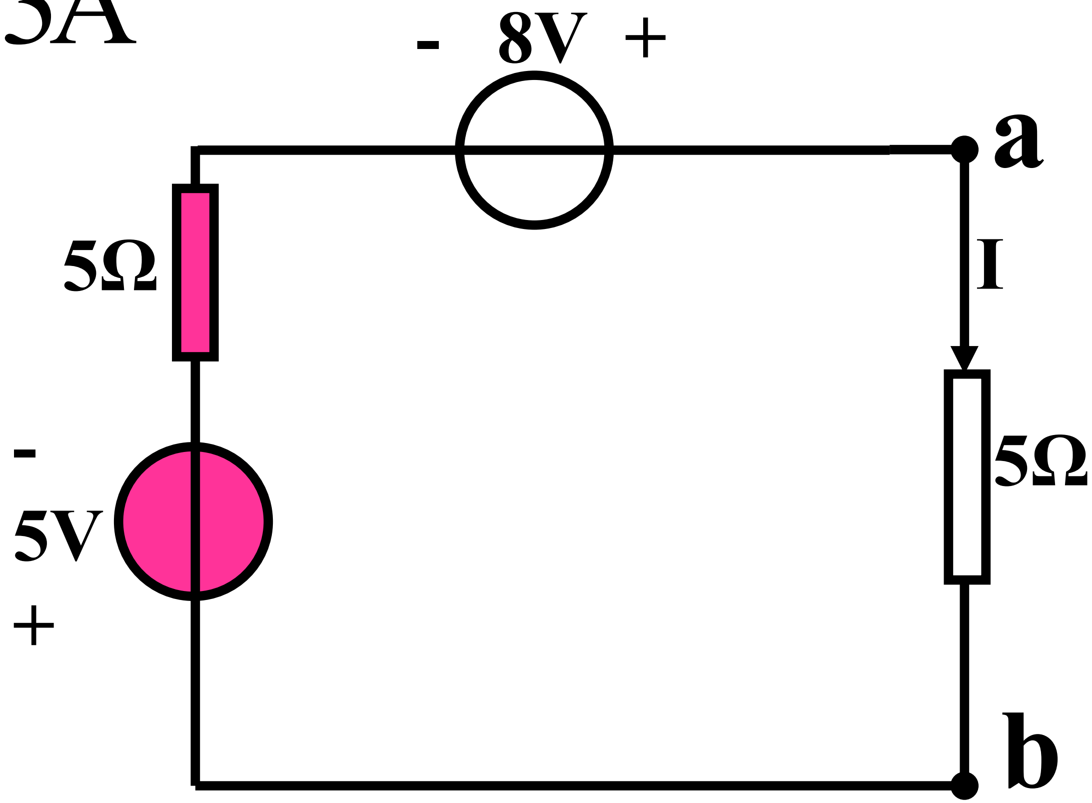


例：求电流I.

$$5+5I-8+5I=0$$

$$I=0.3\text{A}$$





小结

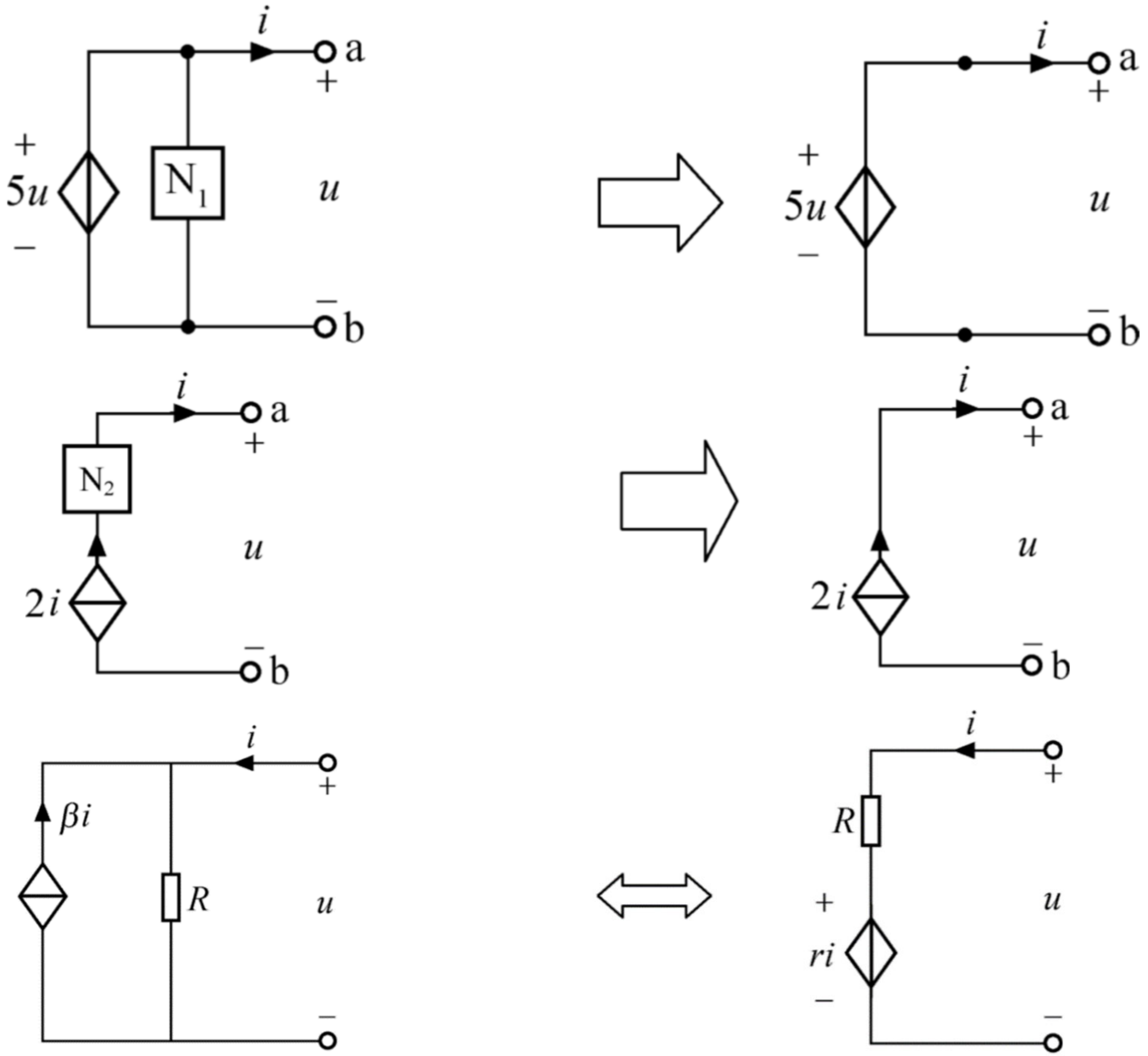
- 作用：电路等效变换
- 对象：有内阻R_s的实际电源
- 推广：可把外接电阻看作内阻
- 注意：等效端子

含受控源电路的等效变换

变换规则

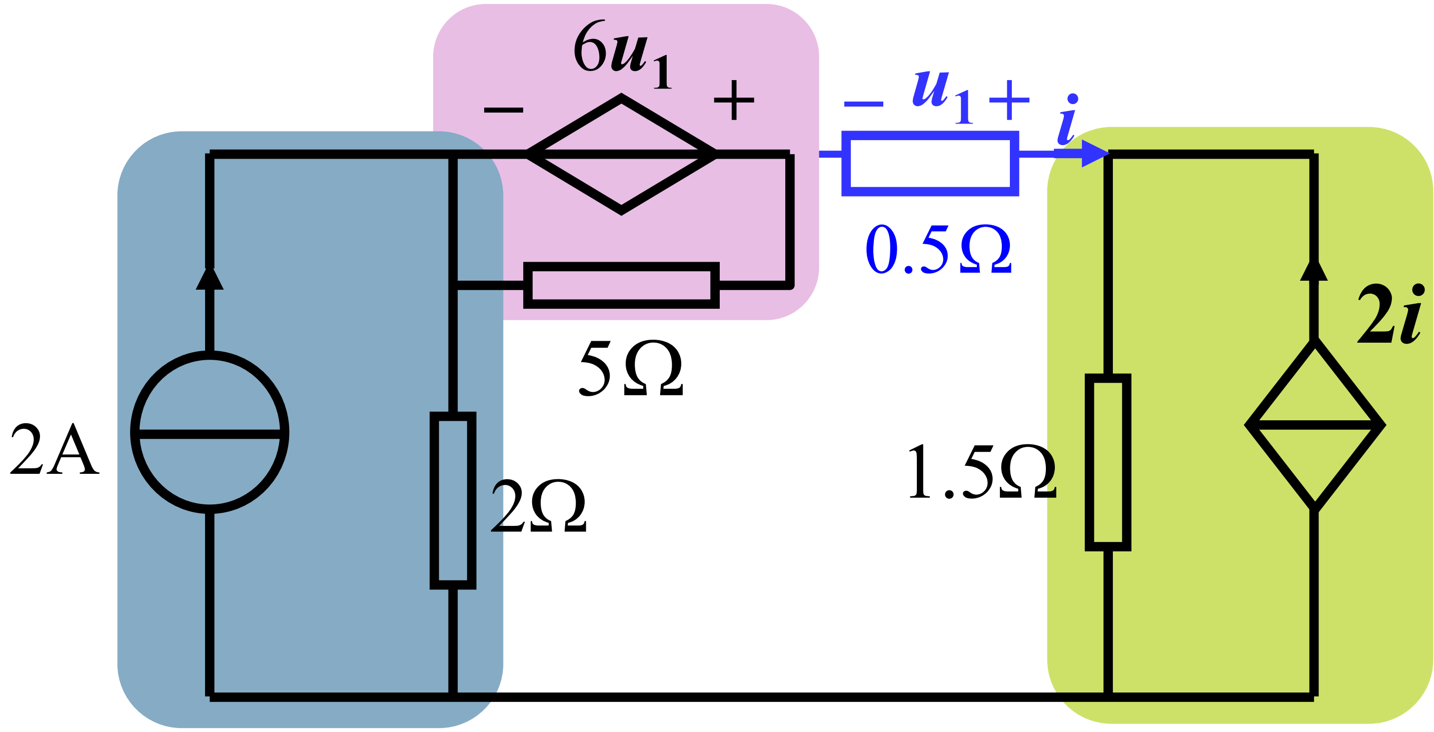
- 与独立源一样处理
- 等效变换时受控源的控制量不能消失

◆含受控源电路的等效变换

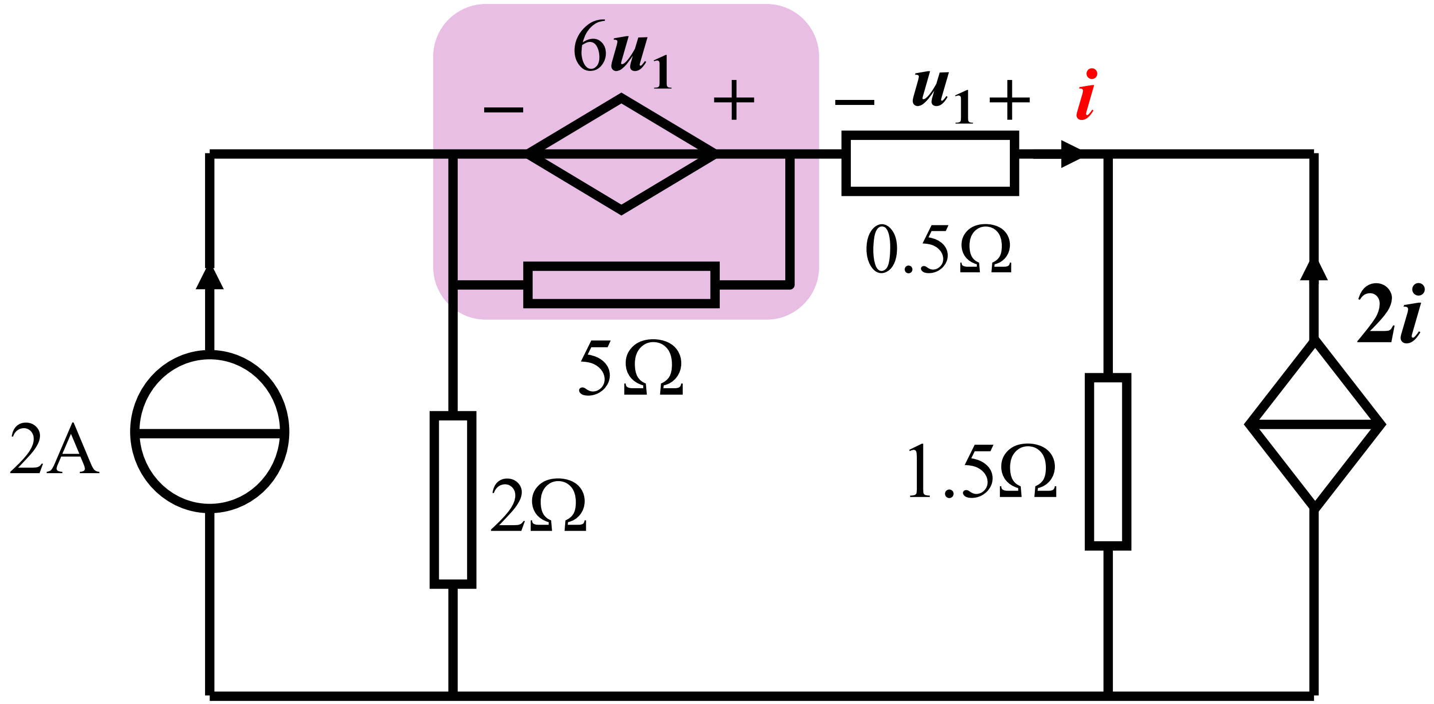


$\beta i * R = \gamma i$

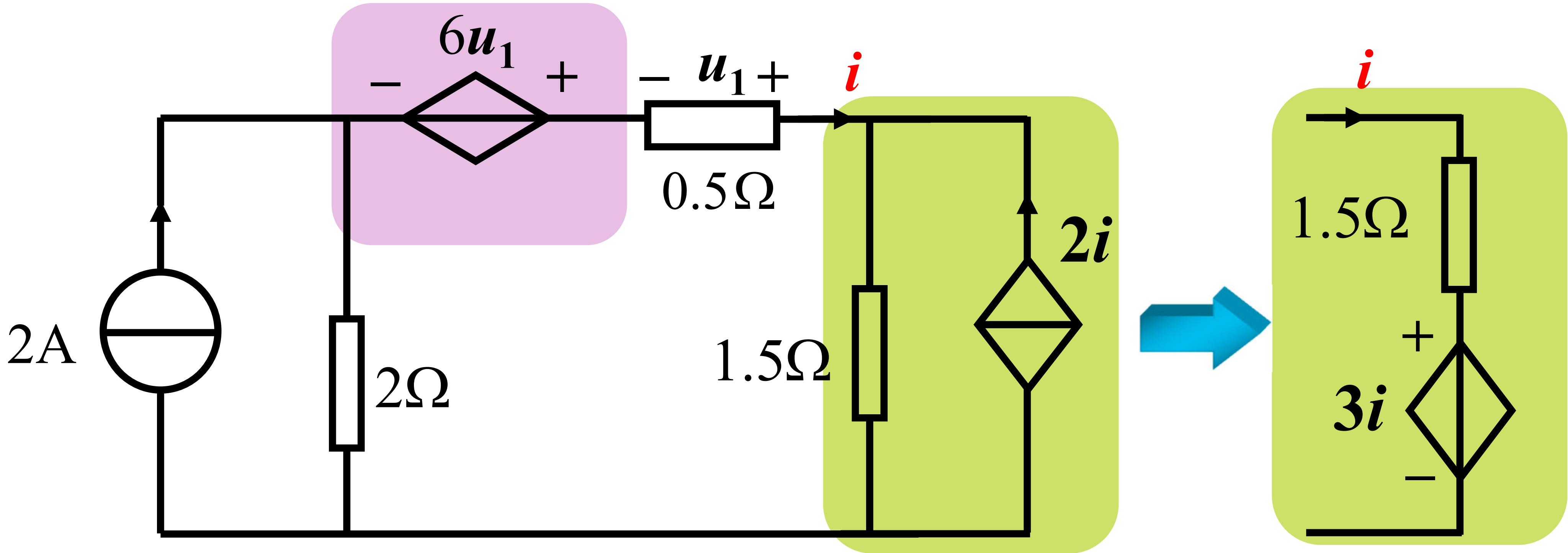
【例】求电流 i



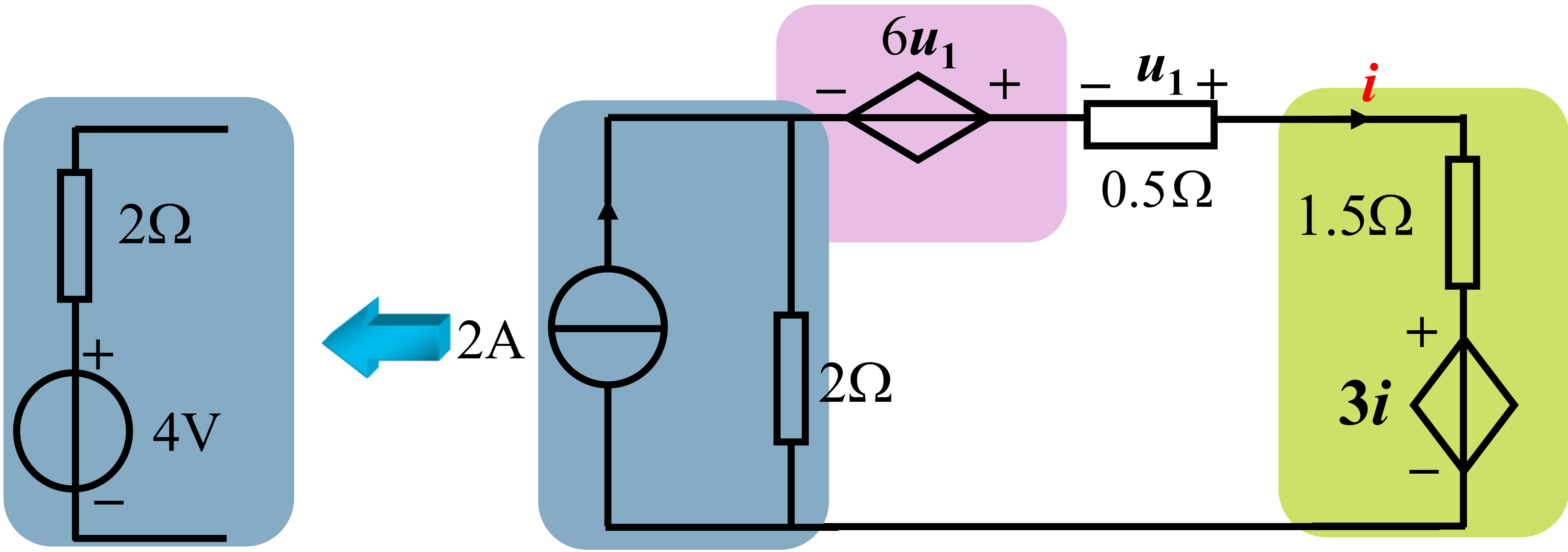
【例】（P40例2-12）求电流 i



【例】（P40例2-12）求电流 i



【例】（P40例2-12）求电流 i



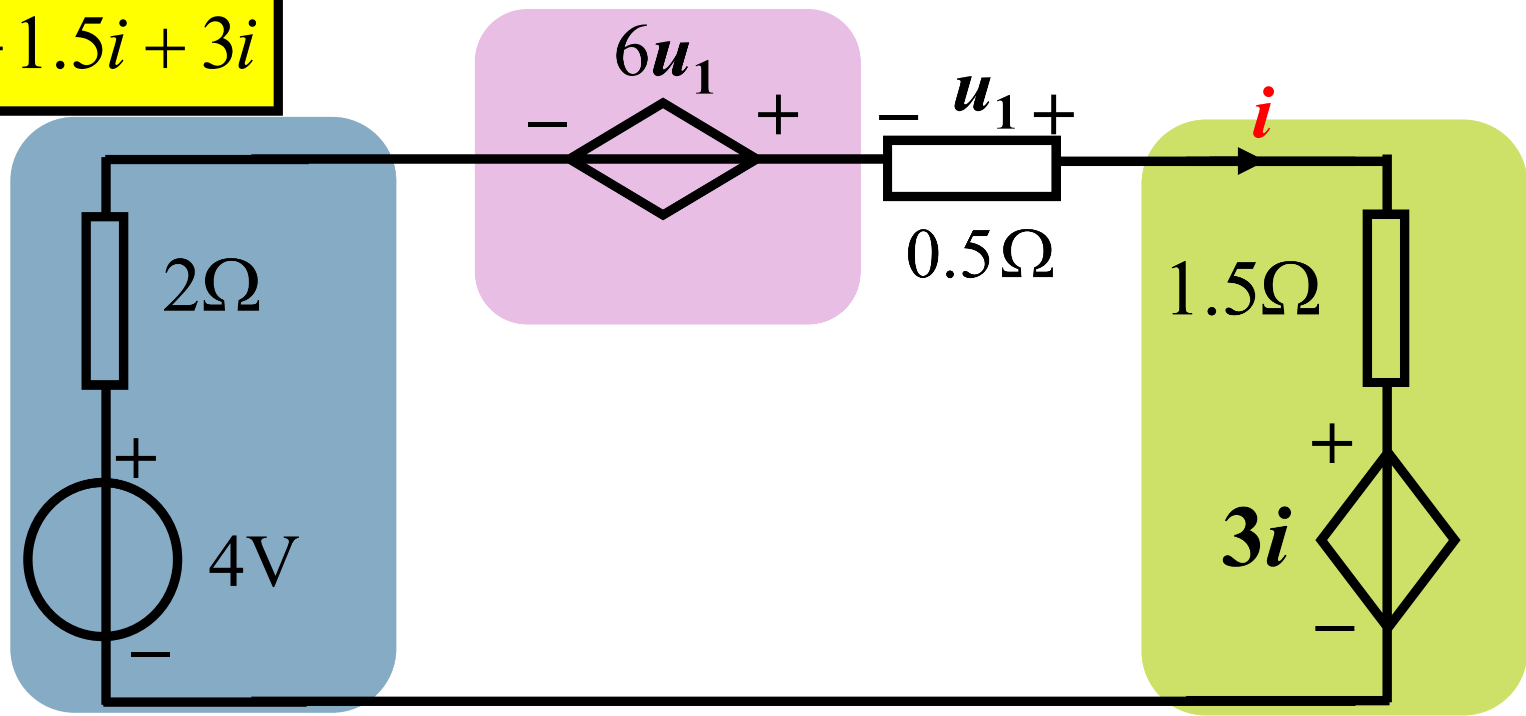
【例】（P40例2-12）求电流 i

$$4 = 2i - 6u_1 - u_1 + 1.5i + 3i$$

KVL

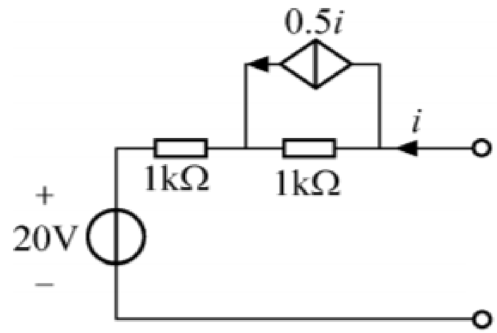
VCR

$$u_1 = -0.5i$$



得：$i = 0.4\text{A}$

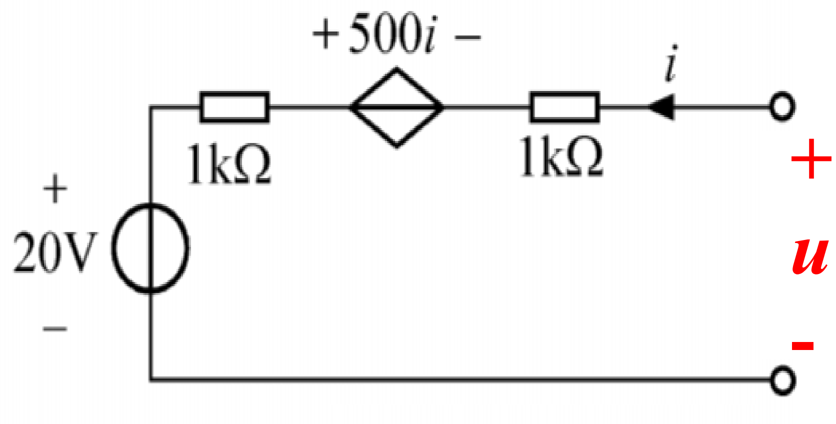
化简如图所示电路：



解：受控源的诺顿模型化为戴维南模型：

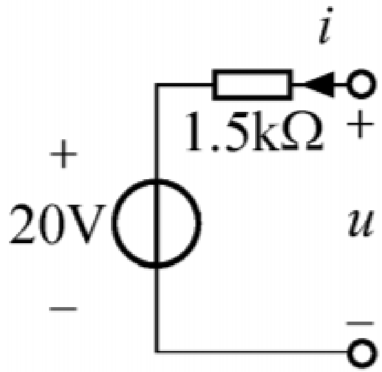
加压求流法：

列些端口电压电流VCR，进而求得等效电路。



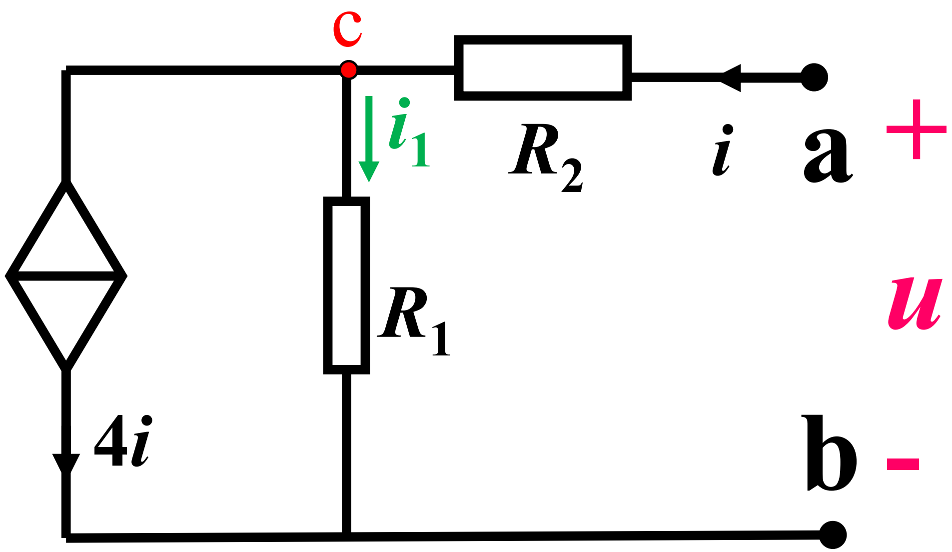
$$u-20-1000i+500i-1000i=0$$

$$u=1500i+20$$



任意线性有源（独立源）二端网络最终都可以等效成一个独立电压源和一个电阻相串联的电路。（戴维南定理）

例 求等效电阻 R_{ab},R_1R_2已知。



解：端口加电压u.

$$R_{ab}=\frac{u}{i}$$

右边网孔列写KVL

$$u-4R_1i_1-R_2i=0$$

C节点列些KCL：

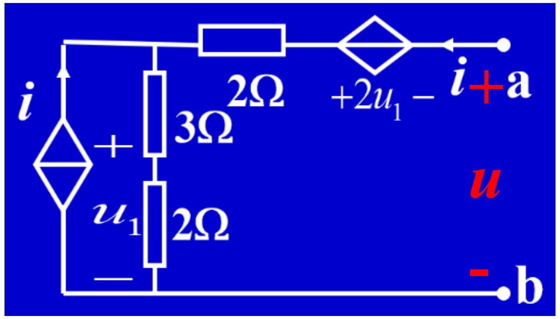
$$i_1+4i-i=0$$

联立求解

$$R_{ab}=\frac{u}{i}=R_2-3R_1$$

仅含有线性受控源及电阻的电路最终等效成一个电阻（可正可负）。受控源电阻性

例 求等效电阻 *R*ab



解：端口加电压u .列端口VCR

$$\begin{cases} u = -2u_1 + 2i + (3+2)(i+i) \\ u_1 = (i+i)\times 2 \end{cases}$$

$$R_{ab} = \frac{u}{i} = 4\Omega$$