电路的网络函数和频率特性

一、网络函数

电路在频率为 ω 的正弦激励下，正弦稳态响应相量与激励相量之比，记为$H(\mathrm{j}\omega)$。

$$H(\mathrm{j}\omega)=\frac{\text{响应相量}}{\text{激励相量}}=\frac{\dot{Y}}{\dot{X}}=\frac{Y\angle\theta_{\mathrm{Y}}}{X\angle\theta_{\mathrm{X}}}=\frac{Y}{X}\angle\theta_{\mathrm{Y}}-\theta_{\mathrm{X}}$$

- $H(\mathrm{j}\omega)$描述了激励相量为$1\angle 0°$时响应相量随频率的变化。

极坐标：$H(\mathrm{j}\omega)=|H(\mathrm{j}\omega)|\angle\theta(\omega)$

$$|H(\mathrm{j}\omega)|=\frac{Y}{X}\qquad \angle\theta(\omega)=\angle\theta_{Y}-\theta_{X}$$

二、频率特性

电路响应随激励频率而变的特性。 $H(\mathrm{j}\omega)=|H(\mathrm{j}\omega)|\angle\theta(\omega)$

$$|H(\mathrm{j}\omega)|=\frac{Y}{X}$$

响应与激励的幅值比

$$\angle\theta(\omega)=\angle\theta_Y-\theta_X$$

响应与激励的相位差

幅频特性——振幅比$|H(\mathrm{j}\omega)|$随ω的变化特性；

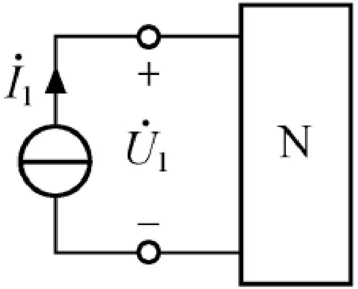
相频特性——相位$\theta(\omega)$随ω的变化特性。

振幅比或相位作纵坐标，频率为横坐标的曲线,分别称为网络函数的幅频特性曲线和相频特性曲线。

三、网络函数的分类

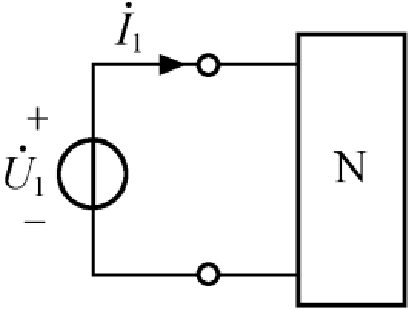
1.策动点函数 输入和输出属于同一端口。

（1）策动点阻抗：$Z(\mathrm{j}\omega)=\dfrac{\dot{U}_1}{\dot{I}_1}$



（2）策动点导纳：$Y(\mathrm{j}\omega)=\dfrac{\dot{I}_1}{\dot{U}_1}$

$$Z(\mathrm{j}\omega)=\frac{1}{Y(\mathrm{j}\omega)}$$

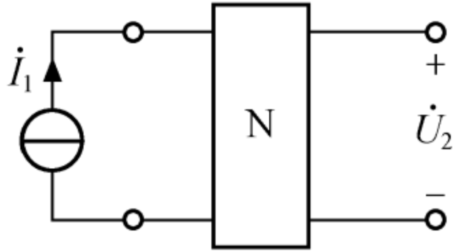


2.转移函数/传输函数

输入和输出属于不同端口

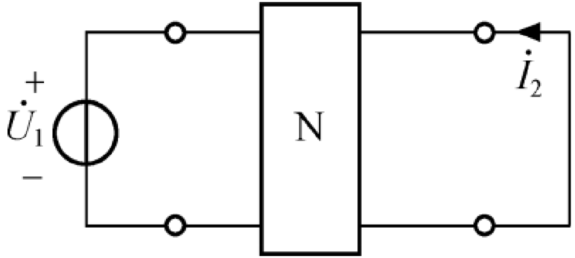
（3）转移阻抗：

$$Z_{\mathrm{T}}(\mathrm{j}\omega)=\frac{\dot{U}_2}{\dot{I}_1}$$



（4）转移导纳：

$$Y_{\mathrm{T}}(\mathrm{j}\omega)=\frac{\dot{I}_2}{\dot{U}_1}$$

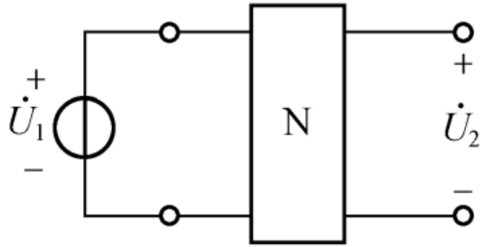


转移阻抗和转移导纳之间不存在互为倒数的关系。

2.转移函数/传输函数

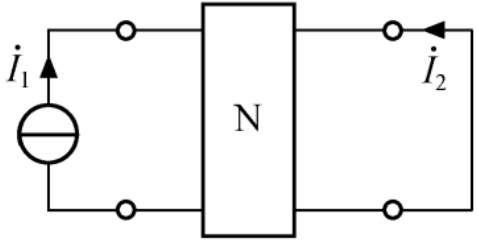
（5）转移电压比：

$$K_u(j\omega)=\frac{\dot{U}_2}{\dot{U}_1}$$

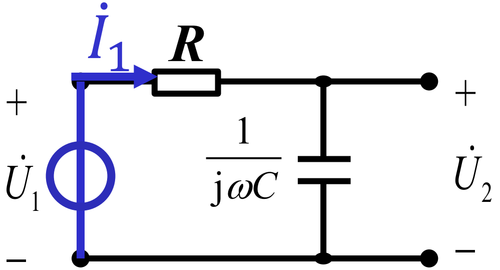


(6) 转移电流比：

$$\boldsymbol{K}_I(j\omega)=\frac{\dot{\boldsymbol{I}}_2}{\dot{\boldsymbol{I}}_1}$$



【例1】试求图示网络负载端开路时的网络函数$H(\mathrm{j}\omega) = \dot{U}_2 / \dot{U}_1$。



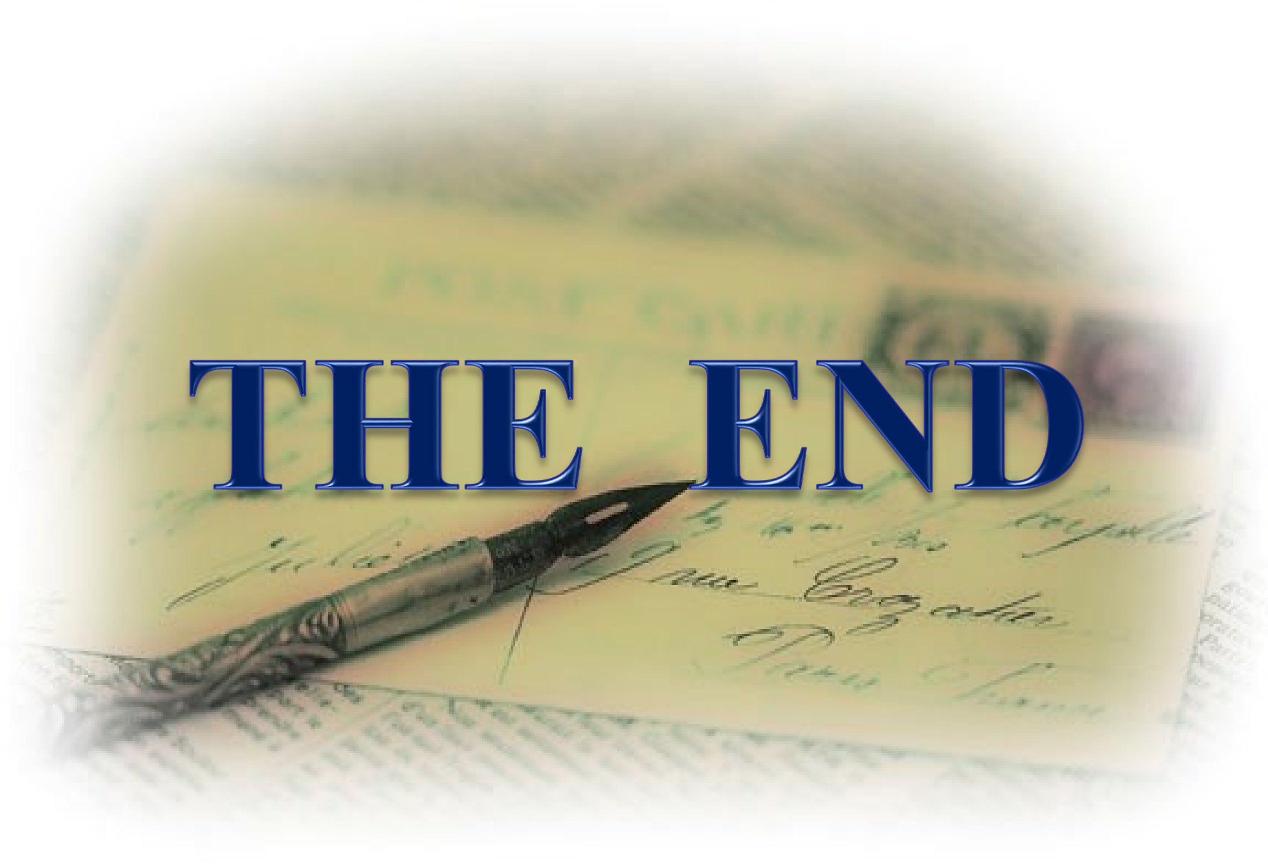
解：步骤：1.以输入电压（电流）作为已知。2.用正弦稳态分析方法求输出相量。3.然后与输入相量相比。

$$\dot{U}_2 = \frac{\frac{1}{\mathrm{j}\omega C}}{R + \frac{1}{\mathrm{j}\omega C}} \dot{U}_1 = \frac{1}{1 + \mathrm{j}\omega RC} \dot{U}_1$$

若求转移阻抗$H(\mathrm{j}\omega) = \dot{U}_2 / \dot{I}_1$?

则：$H(\mathrm{j}\omega) = \frac{\dot{U}_2}{\dot{U}_1} = \frac{1}{1 + \mathrm{j}\omega RC}$

- $H(\mathrm{j}\omega)$取决于网络的结构和参数，与输入无关。
- $H(\mathrm{j}\omega)$ 是以ω为变量的函数。



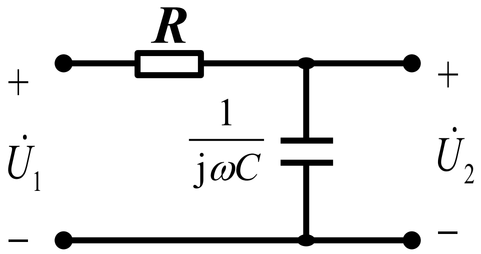
THE END

RC电路的频率特性

一、RC低通网络

转移电压比

$$K_U(j\omega)=\frac{\dot U_2}{\dot U_1}=\frac{\dfrac{1}{j\omega C}}{R+\dfrac{1}{j\omega C}}=\frac{1}{1+j\omega RC}$$



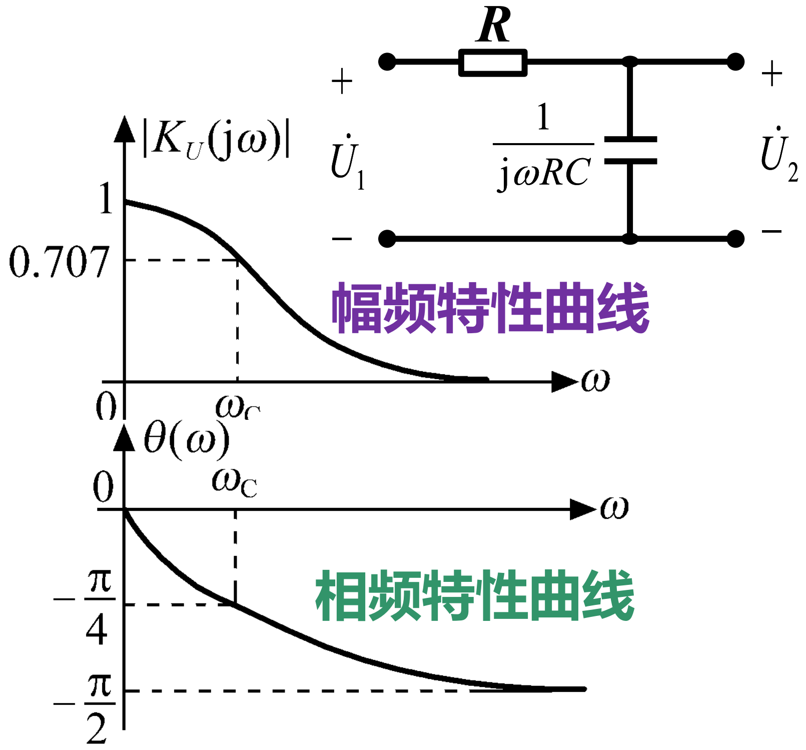
令 $\omega_C=\dfrac{1}{RC}$ $\quad K_U(j\omega)=\dfrac{1}{1+j\dfrac{\omega}{\omega_C}}$

$$\left|K_U(j\omega)\right|=\frac{1}{\sqrt{1+\left(\dfrac{\omega}{\omega_C}\right)^2}}$$ 幅频特性

$$\theta(\omega)=-\arctan\frac{\omega}{\omega_C}$$ 相频特性

$$|K_U(\mathrm{j}\omega)| = \frac{1}{\sqrt{1+\left(\dfrac{\omega}{\omega_C}\right)^2}}$$ 幅频特性

$$\theta(\omega) = -\arctan\frac{\omega}{\omega_C}$$ 相频特性



当$\omega=0$**时，** $|K_U(\mathrm{j}\omega)|=1$**，** $\theta(\omega)=0$

当$\omega=\omega_C$**时，** $|K_U(\mathrm{j}\omega)|=1/\sqrt{2}$ $\theta(\omega)=-\pi/4$

当$\omega\to\infty$**时，** $|K_U(\mathrm{j}\omega)|\to 0, \theta(\omega)\to -\pi/2$

截止频率：幅频特性的值$|H(\mathrm{j}\omega)|$降至最大值的0.707倍时的频率。又称半功率频率。

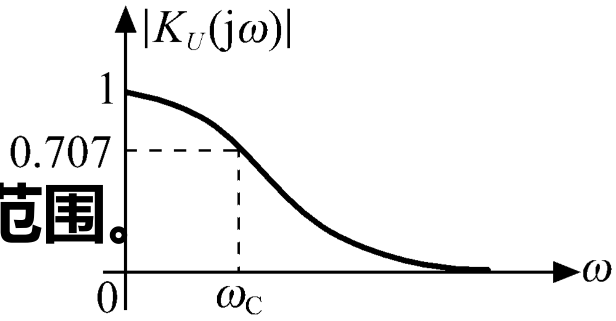
通频带(通带)：幅频特性的值从最大值下降到0.707 的频率范围。此例：0~ ω_C。ω_C称为**截止频率**。

带宽：通频带宽度。

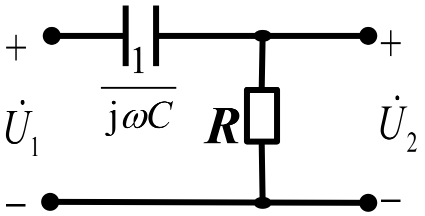
$\omega_C - 0 = \omega_C$

阻带：幅频特性最大值的0.707 以下频率范围。

$\omega > \omega_C$。



二、RC高通网络



转移电压比

$$K_U(\mathrm{j}\omega)=\frac{\dot{U}_2}{\dot{U}_1}=\frac{R}{R+\frac{1}{\mathrm{j}\omega C}}=\frac{1}{1+\frac{1}{\mathrm{j}\omega RC}}$$

令 $\omega_C=\frac{1}{RC}$ 　 $K_U(\mathrm{j}\omega)=\frac{1}{1+\frac{\omega_C}{\mathrm{j}\omega}}$

$$\left|K_U(\mathrm{j}\omega)\right|=\frac{1}{\sqrt{1+\left(\frac{\omega_C}{\omega}\right)^2}}$$

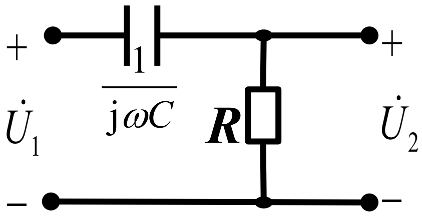
幅频特性

$$\theta(\omega)=\arctan\frac{\omega_C}{\omega}$$

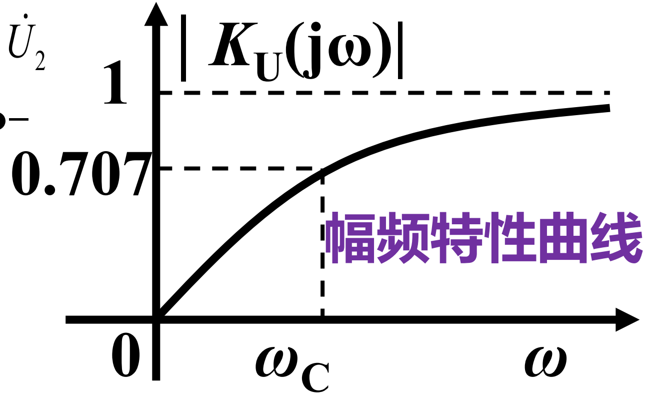
相频特性

◆ RC电路的频率特性

$$\left|K_U(\mathrm{j}\omega)\right| = \frac{1}{\sqrt{1+\left(\dfrac{\omega_C}{\omega}\right)^2}}$$



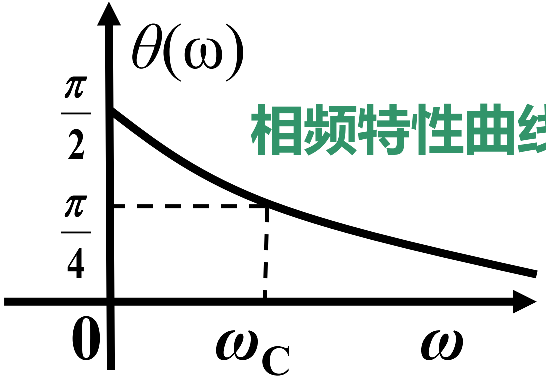
$$\theta(\omega) = \arctan\frac{\omega_C}{\omega}$$



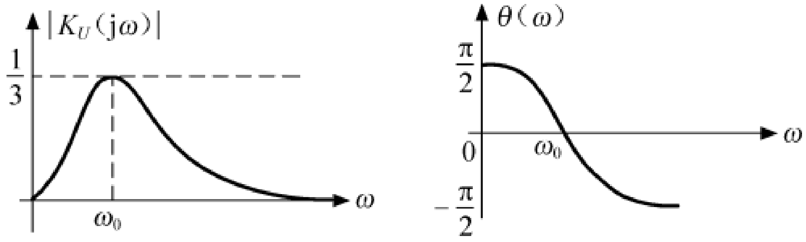
当$\omega=0$**时,** $|K_U(j\omega)|=0, \theta(\omega)=\pi/2$

当$\omega=\omega_C$**时,** $\left|K_U(\mathrm{j}\omega)\right| = 1/\sqrt{2} \quad \theta(\omega)=\pi/4$

当$\omega\to\infty$**时,** $|K_U(j\omega)|\to 1, \theta(\omega)\to 0$

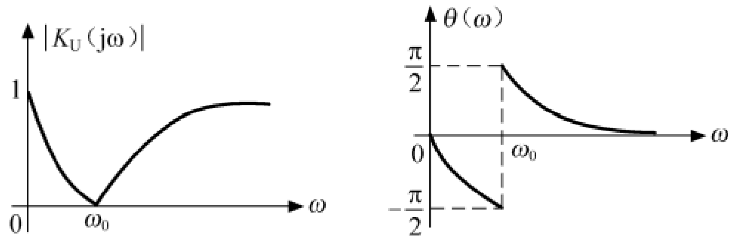


截止频率ω_C



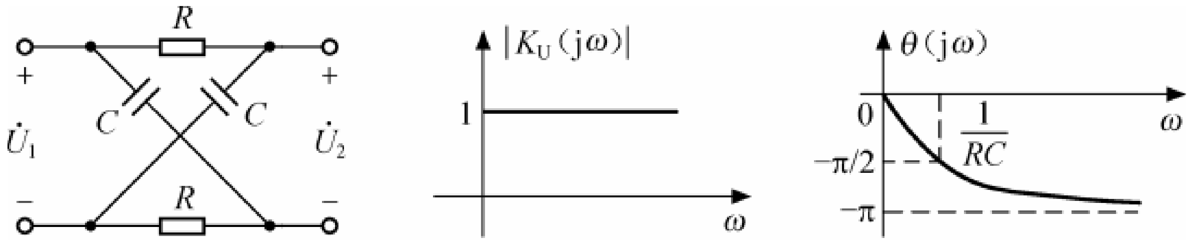
（b）幅频特性　　（c）相频特性

图 10-5　RC 带通网络及其频率特性



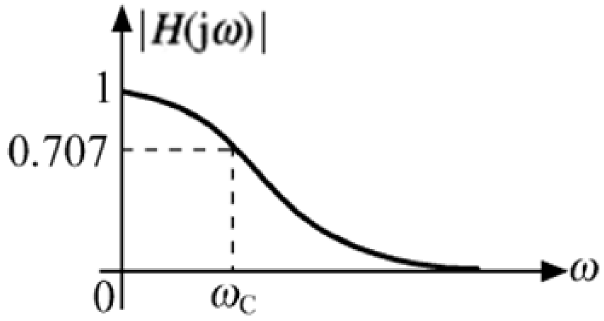
（b）幅频特性　　（c）相频特性

图 10-6　RC 带阻网络及其频率特性

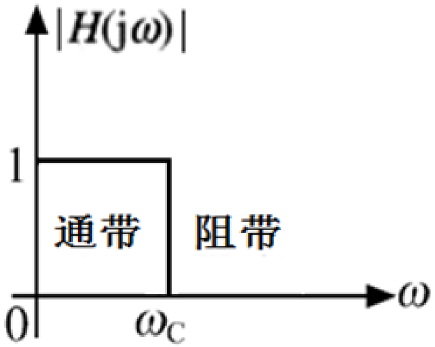


（a）RC 全通网络　　（b）幅频特性　　（c）相频特性

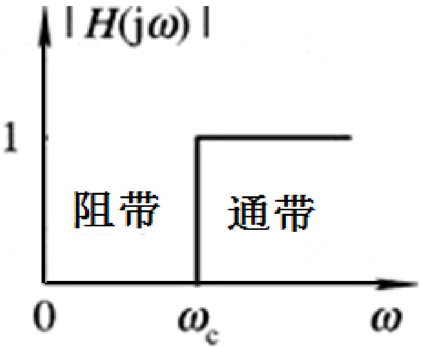
图 10-7　RC 全通网络及其频率特性



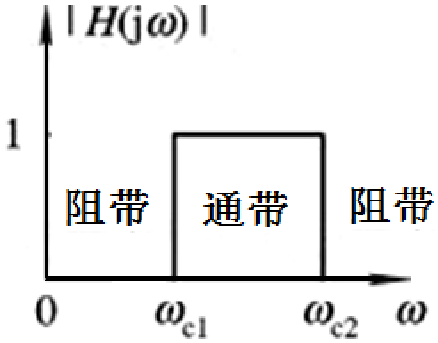
实际低通滤波器



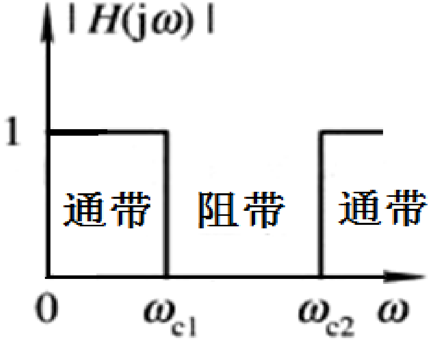
理想低通滤波器



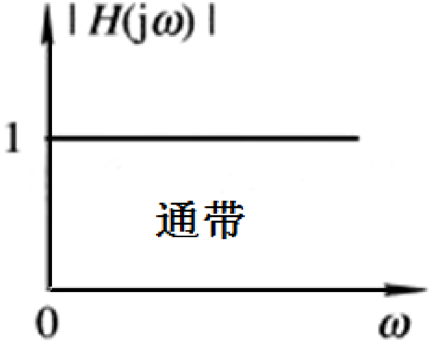
理想高通滤波器



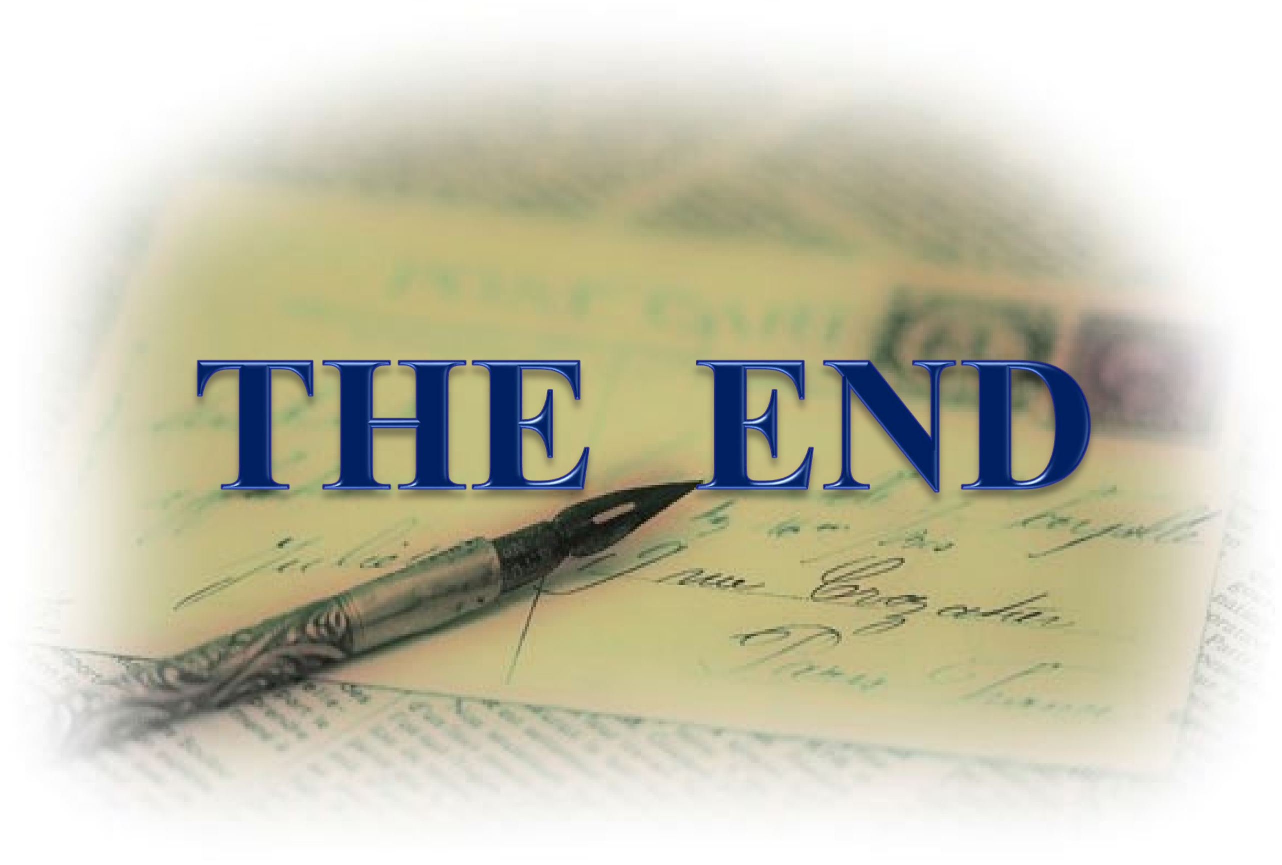
理想带通滤波器



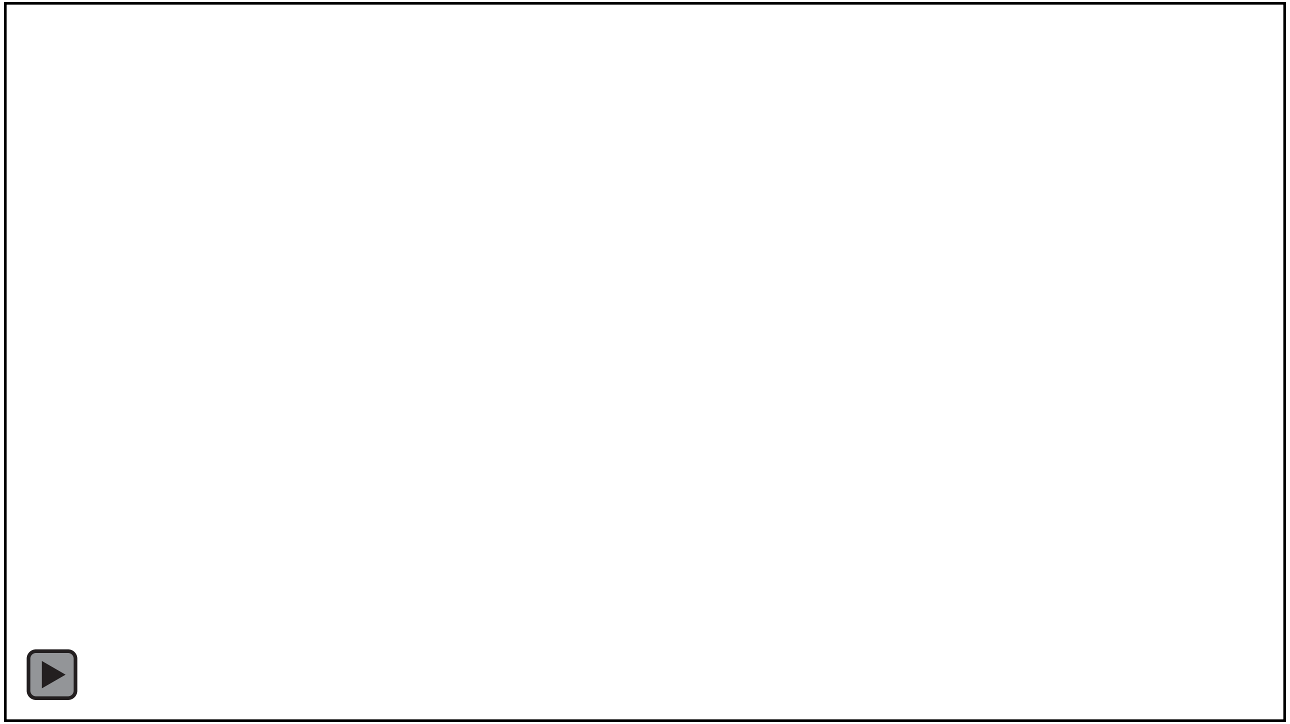
理想带阻滤波器



理想全通滤波器


THE END

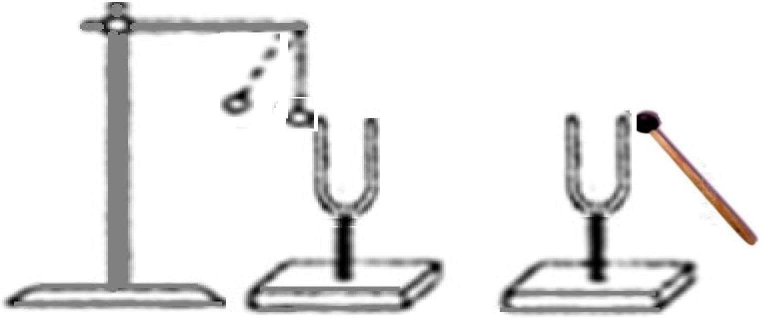
RLC串联谐振电路

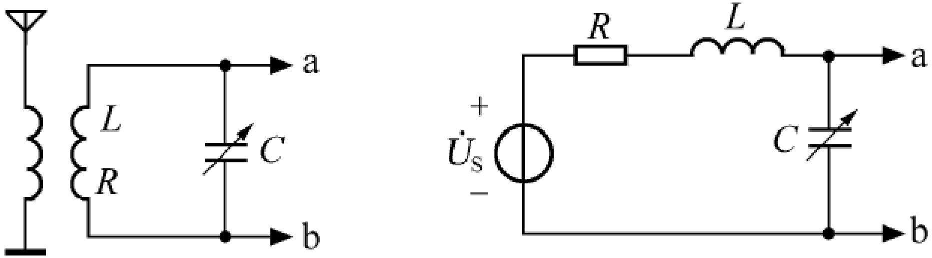


一、 谐振

① 音叉共鸣 （声学） ⟶ 共振频率 发声

② 电路谐振 （电气） ⟶ 谐振频率

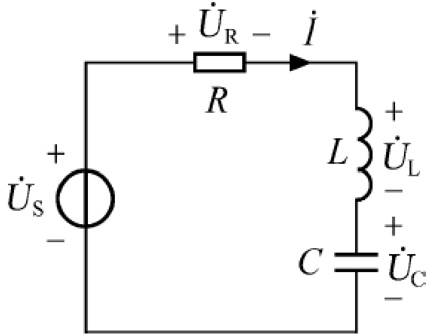




收音机的输入电路及等效电路

端口电压和电流相位相同的情况时， 称电路发生谐振。

二、 RLC电路的谐振条件



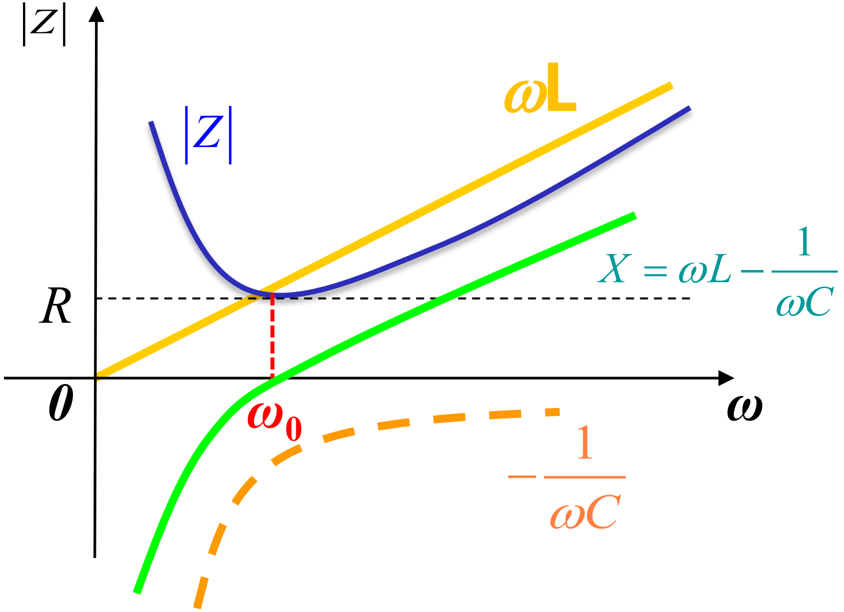
$$Z(j\omega)=\frac{\dot{U}_S}{\dot{I}}$$

$$=R+j(\omega L-\frac{1}{\omega C})$$

$$|Z(j\omega)|=\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}$$

$$\theta_Z=\arctan\left(\frac{\omega L-\frac{1}{\omega C}}{R}\right)$$

阻抗幅频特性图



二、 RLC电路的谐振条件

$$\theta_Z = \arctan\left(\frac{\omega L - \dfrac{1}{\omega C}}{R}\right)$$

$$X = \omega L - \frac{1}{\omega C} = 0$$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

$$f = f_0 = \frac{1}{2\pi\sqrt{LC}}$$

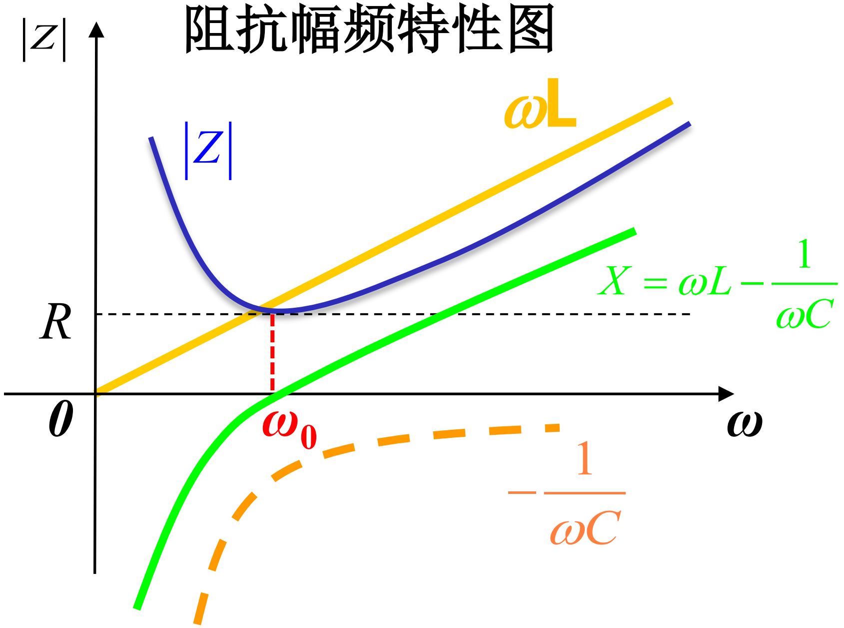
固有谐振(角)频率

电路已定，调输入信号频率

输入信号频率已定，调电路

$$|Z(j\omega)| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

阻抗幅频特性图



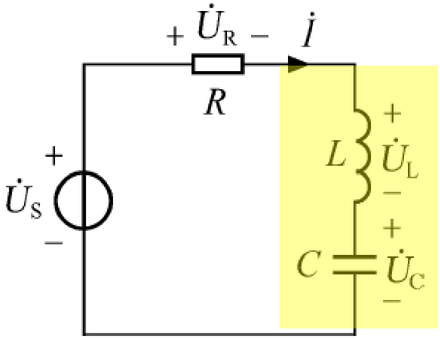
$\omega < \omega_0$ 容性

$\omega > \omega_0$ 感性

三、 RLC电路的谐振特性

谐振时的阻抗

$$|Z(j\omega)| = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$



$$Z(j\omega_0) = Z_0 = R$$

$$|Z(j\omega)|_{min} = |Z(j\omega_0)| = R$$

特征阻抗

$$\rho = \omega_0 L = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}}$$

谐振时的电流

$$\omega = \omega_0,\quad \dot{I}_0 = \frac{\dot{U}_S}{Z_0} = \frac{\dot{U}_S}{R}$$

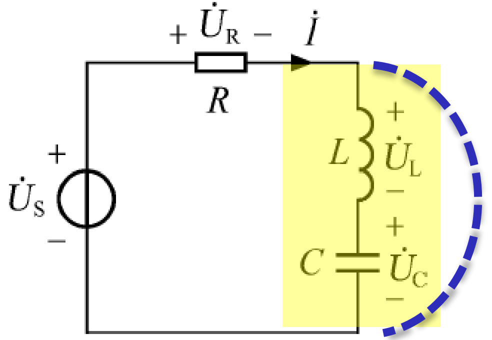
品质因数

$$Q = \frac{\rho}{R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$|\dot{I}|_{max} = \left|\frac{\dot{U}_S}{Z}\right|_{max} = \frac{U_s}{|Z|_{min}}$$

三、 RLC电路的谐振特性

谐振时的电压

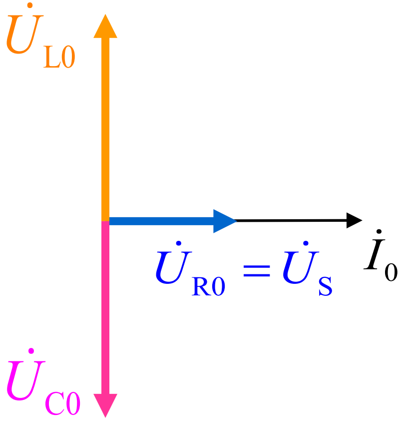


$$\dot{U}_{R0} = R\dot{I}_0 = \dot{U}_S$$

$$\dot{U}_{L0} = j\omega_0 L\dot{I}_0 = j\frac{\omega_0 L}{R}\dot{U}_S = jQ\dot{U}_S$$

$$\dot{U}_{C0} = \frac{1}{j\omega_0 C}\dot{I}_0 = -j\frac{1}{\omega_0 RC}\dot{U}_S = -jQ\dot{U}_S$$

$$\dot{U}_{C0} + \dot{U}_{L0} = 0$$



通常

$Q>>1, U_{L0}=U_{C0}=QU_S>>U_S=U_R$

___**电压谐振**

谐振时的能量

$$i_0 = \sqrt{2} I_0 \cos \omega_0 t$$

$$\dot{I}_0 = I_0 \angle 0^\circ \qquad \dot{U}_{C0} = \frac{1}{j\omega_0 C} \dot{I}_0 = \frac{I_0}{\omega_0 C} \angle -90^\circ$$

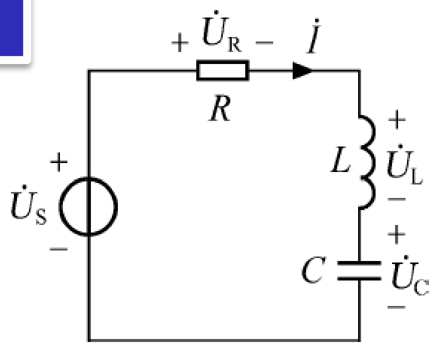
$$u_{C0} = \sqrt{2} \frac{I_0}{\omega_0 C} \cos(\omega_0 t - 90^\circ) = \sqrt{2} \frac{I_0}{\omega_0 C} \sin \omega_0 t$$

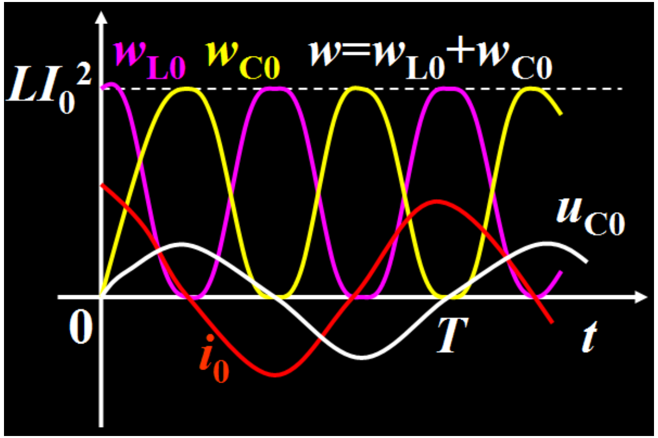
$$w_{L0}(t) = \frac{1}{2} L i_{L0}^2 = L I_0^2 \cos^2 \omega_0 t$$

$$w_{C0}(t) = \frac{1}{2} C u_{C0}^2 = C \left(\frac{I_0}{\omega_0 C} \right)^2 \sin^2 \omega_0 t$$

$$= C \left(\frac{I_0}{\frac{1}{\sqrt{LC}} C} \right)^2 \sin^2 \omega_0 t = L I_0^2 \sin^2 \omega_0 t$$

$$w = w_{L0} + w_{C0} = L I_0^2 = C U_{C0}^2$$



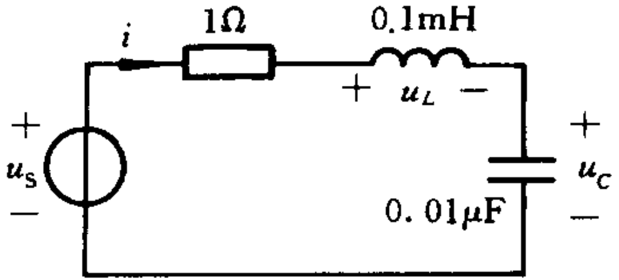


储能元件与电源无能量交换

【例】 电路如图，已知$u_S(t)=\sqrt{2}\cos\omega t$ mV。

求：(1) 频率ω为何值时，电路发生谐振。

(2) 电路谐振时，求 $\dot{U}_{L0}$ $\dot{U}_{C0}$及其有效值。



解：(l)电压源的角频率应为

$$\omega=\omega_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{10^{-4}\times10^{-8}}}=10^6\,\text{rad/s}$$

(2)品质因数: $Q=\omega_0 L/R=100$

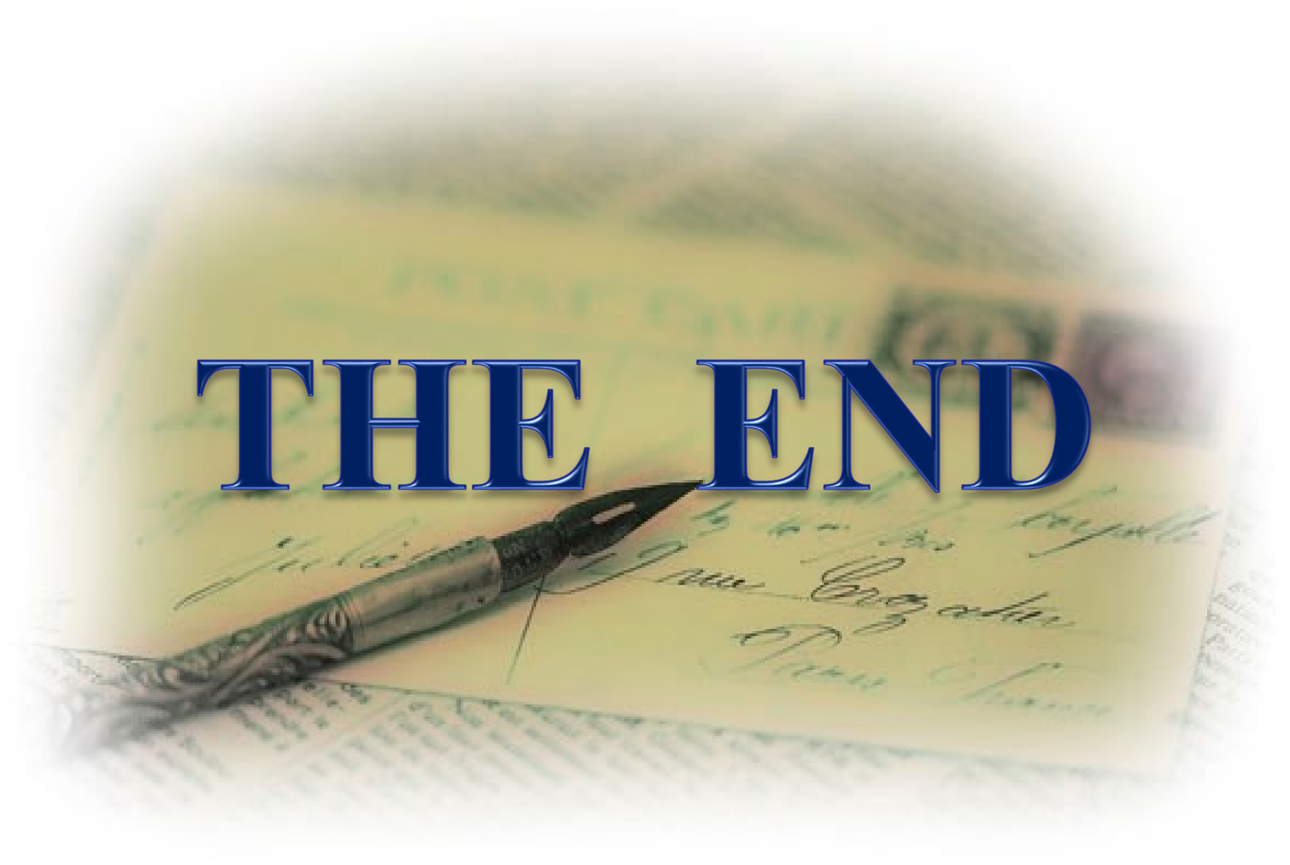
$\dot{U}_S=1\angle 0^\circ$

$\dot{U}_{L0}=\mathrm{j}Q\dot{U}_S=100\angle 90^\circ\,\text{mV}$

$U_{L0}=100\,\text{mV}$

$\dot{U}_{C0}=-\mathrm{j}Q\dot{U}_S=100\angle -90^\circ\,\text{mV}$

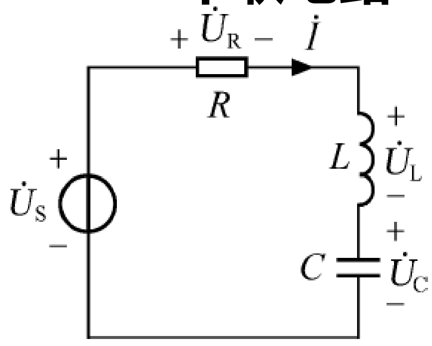
$U_{C0}=100\,\text{mV}$


THE END

RLC串联电路频率特性

一、RLC串联电路电流的频率特性

RLC串联电路



$$Y(j\omega)=\frac{\dot{I}}{\dot{U}_S}=\frac{1}{R+j\omega L+\dfrac{1}{j\omega C}}=\frac{1/R}{1+j\left(\dfrac{\omega L}{R}-\dfrac{1}{\omega RC}\right)}$$

$$=\frac{Y_0}{1+jQ\left(\dfrac{\omega}{\omega_0}-\dfrac{\omega_0}{\omega}\right)}$$

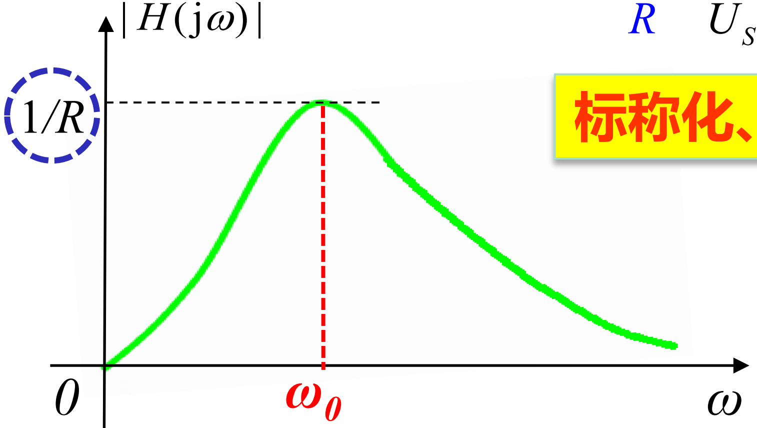
$$(\because Q=\frac{\omega_0 L}{R}=\frac{1}{\omega_0 RC})$$

$$(\because \frac{1}{R}=\frac{\dot{I}_0}{\dot{U}_S}=Y_0)$$

标称化、归一化

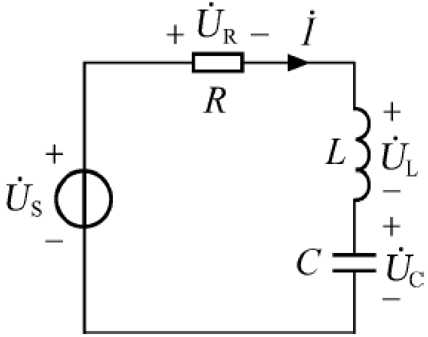
$$|H(j\omega)|=\frac{1/R}{\sqrt{1+Q^2\left(\dfrac{\omega}{\omega_0}-\dfrac{\omega_0}{\omega}\right)^2}}$$

$$\theta(\omega)=-\arctan Q\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)$$



幅频特性曲线

RLC串联电路



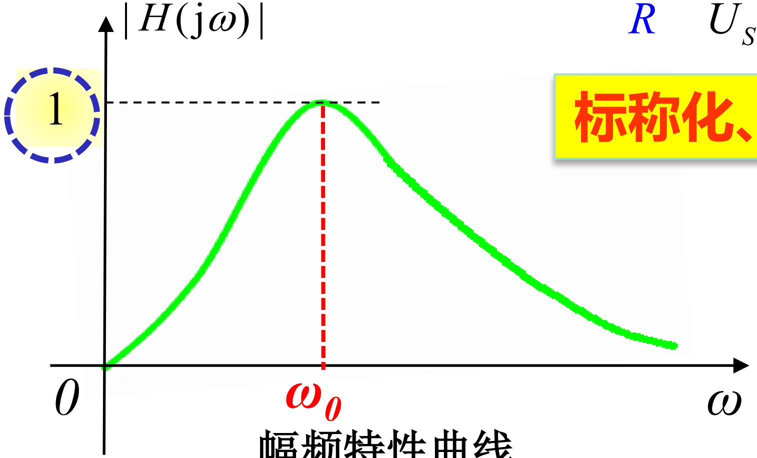
$$Y(\mathrm{j}\omega)=\frac{\dot{I}}{\dot{U}_S}=\frac{1}{R+\mathrm{j}\omega L+\dfrac{1}{\mathrm{j}\omega C}}=\frac{1/R}{1+\mathrm{j}\left(\dfrac{\omega L}{R}-\dfrac{1}{\omega RC}\right)}$$

$$=\frac{Y_0}{1+\mathrm{j}Q\left(\dfrac{\omega}{\omega_0}-\dfrac{\omega_0}{\omega}\right)}$$

$$(\because Q=\frac{\omega_0 L}{R}=\frac{1}{\omega_0 RC})$$

$$(\because \frac{1}{R}=\frac{\dot{I}_0}{\dot{U}_S}=Y_0)$$

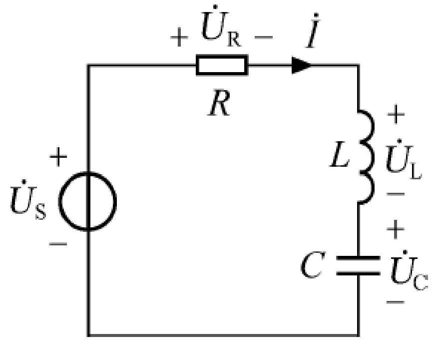
$H(\mathrm{j}\omega)$



幅频特性曲线

标称化、归一化

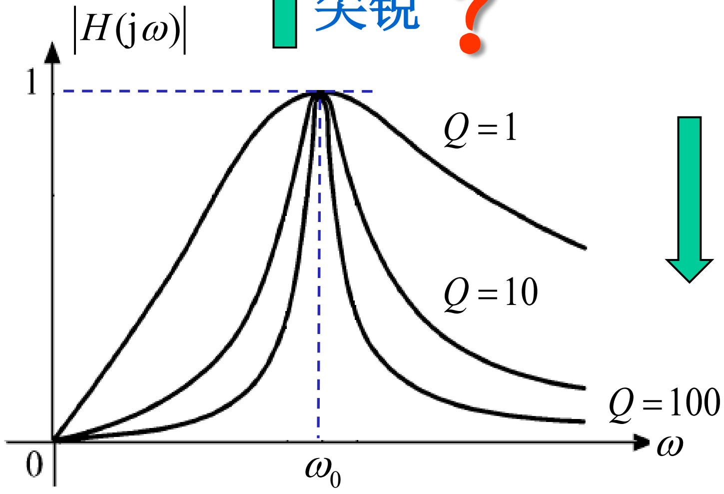
RLC串联电路



$$\left(Q=\frac{\omega_0 L}{R}=\frac{1}{\omega_0 RC}\right)$$

$$\left(\omega_0=\frac{1}{\sqrt{LC}}\right)$$

$$|H(\mathrm{j}\omega)|=\frac{1}{\sqrt{1+Q^2\left(\dfrac{\omega}{\omega_0}-\dfrac{\omega_0}{\omega}\right)^2}}$$



幅频特性曲线

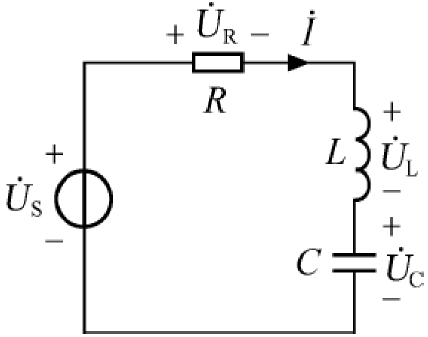
尖锐 ?

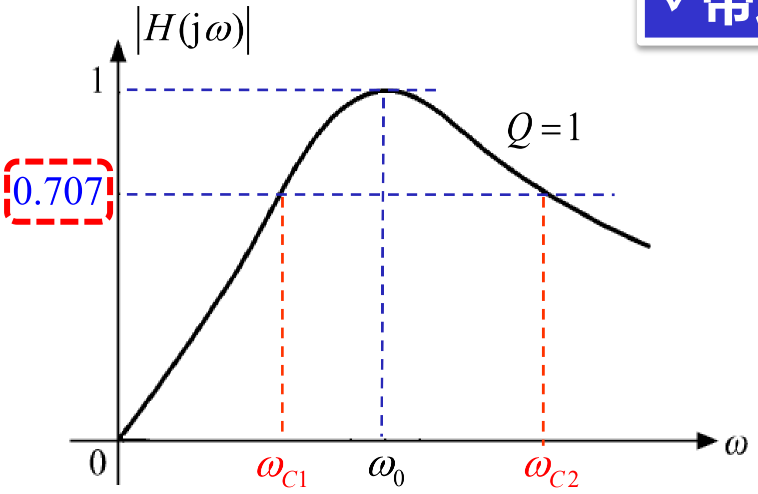
Q值越大

✓Q值和幅频特性曲线的关系?

✓带通滤波器

RLC串联电路





下截止频率　　上截止频率

$$|H(j\omega)| = \frac{1}{\sqrt{1+Q^2\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}}$$

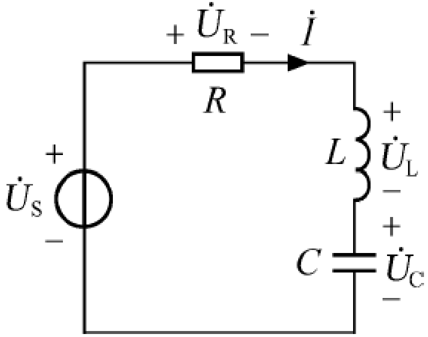
$$Q\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right) = \pm 1$$

$$BW = \Delta\omega = \omega_{C2} - \omega_{C1} = \frac{\omega_0}{Q} = \frac{R}{L}(rad/s)$$

$$BW = \Delta f = f_{C2} - f_{C1} = \frac{f_0}{Q} = \frac{R}{2\pi L}(Hz\text{或}1/s)$$

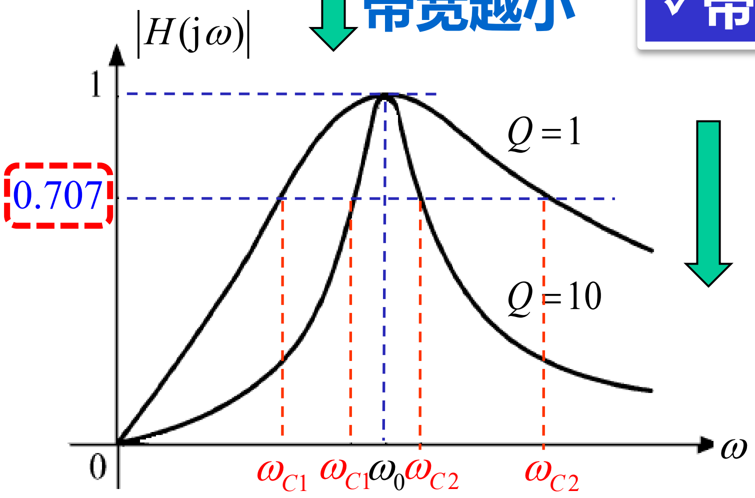
$$\omega_{C1,2} = \omega_0\left(\sqrt{1+\frac{1}{4Q^2}} \pm \frac{1}{2Q}\right)$$

RLC串联电路



↓带宽越小

✓带通滤波器



↓Q值越大

下截止频率　　上截止频率

$$|H(j\omega)| = \frac{1}{\sqrt{1+\boxed{Q^2\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}}}$$

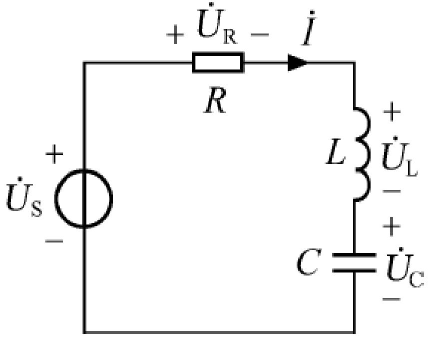
$$\underbrace{}_{=\pm 1}$$

$$\omega_{C1,2} = \omega_0\left(\sqrt{1+\frac{1}{4Q^2}} \pm \frac{1}{2Q}\right)$$

$$BW = \Delta\omega = \omega_{C2}-\omega_{C1} = \boxed{\frac{\omega_0}{Q}} = \frac{R}{L}(rad/s)$$

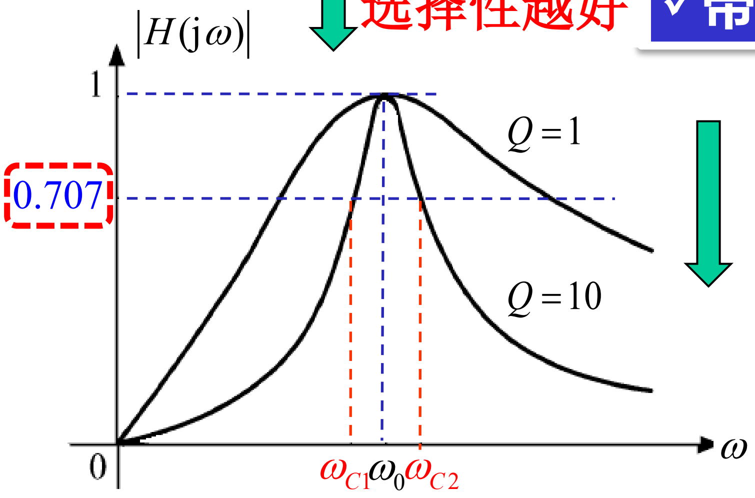
$$BW = \Delta f = f_{C2}-f_{C1} = \frac{f_0}{Q} = \frac{R}{2\pi L}(Hz\text{或}1/s)$$

*RLC*串联电路



选择性越好

✓带通滤波器



Q值越大

下截止频率　　上截止频率

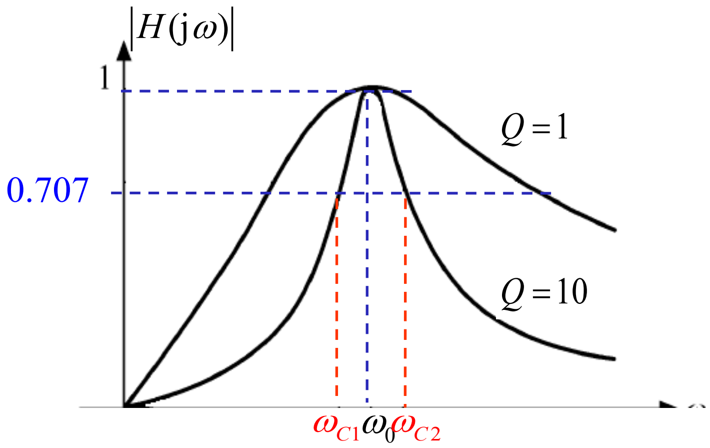
$$|H(j\omega)| = \frac{1}{\sqrt{1+Q^2\underbrace{\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}_{=\pm 1}}}$$

$$BW = \Delta\omega = \omega_{C2}-\omega_{C1} = \boxed{\frac{\omega_0}{Q}} = \frac{R}{L}(rad/s)$$

$$BW = \Delta f = f_{C2}-f_{C1} = \frac{f_0}{Q} = \frac{R}{2\pi L}(Hz\text{或}1/s)$$

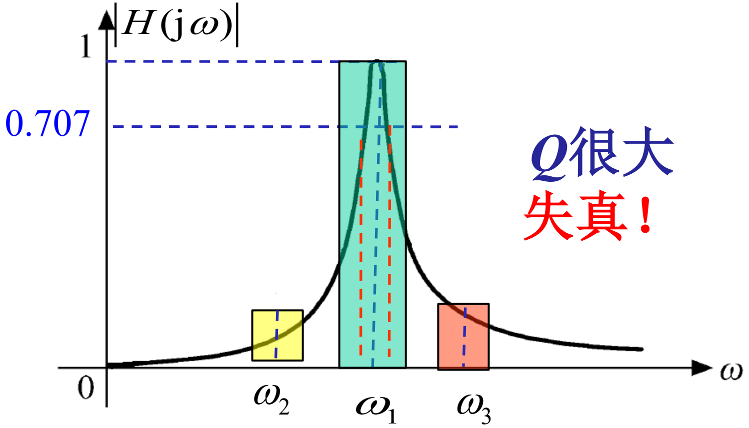
$$\omega_{C1,2} = \omega_0\left(\sqrt{1+\frac{1}{4Q^2}} \pm \frac{1}{2Q}\right)$$

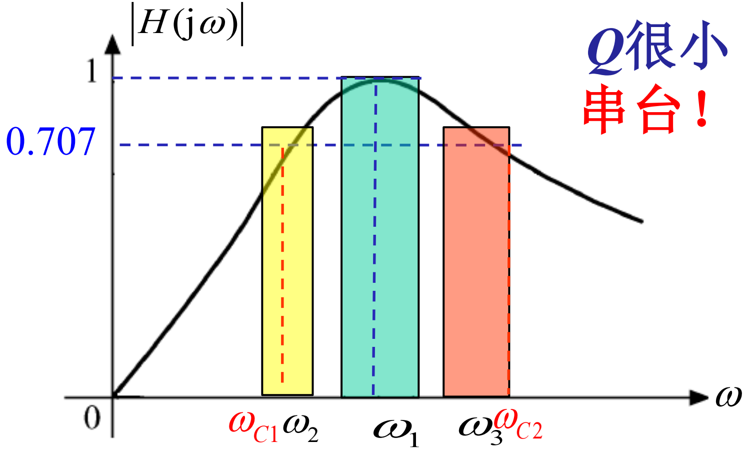
3 带宽与选择性之间的矛盾



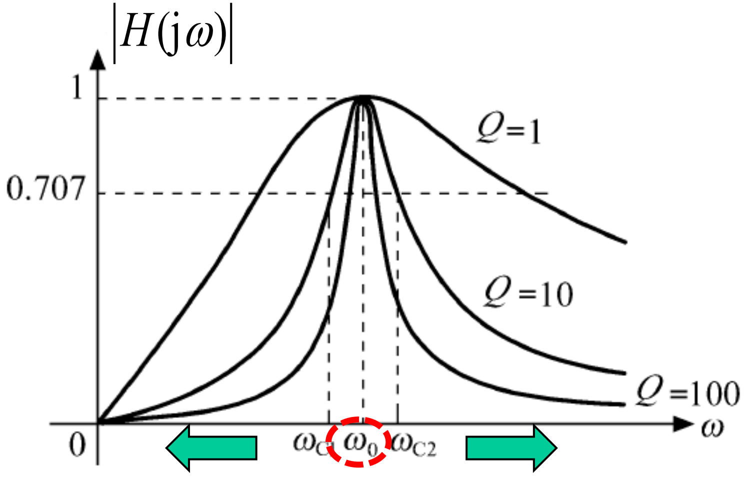
- Q越大，谐振曲线越尖锐，带宽越窄，选择性越好。
- Q小，带宽宽，选择性差

Q要适中

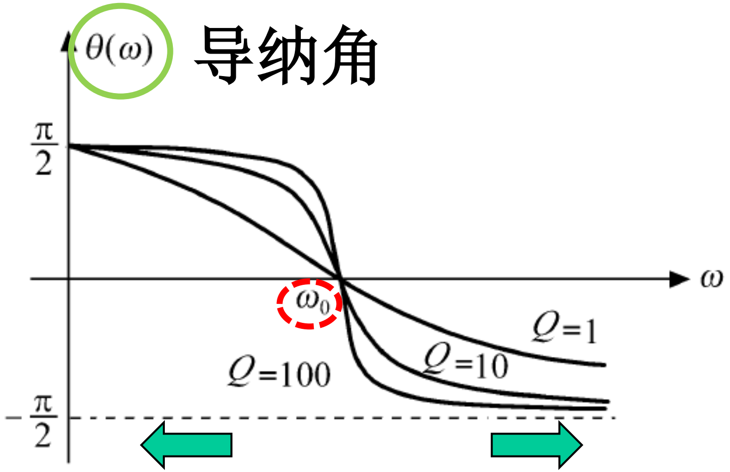




不同Q值下RLC串联电路频率特性



(a) 幅频特性



容性　(b) 相频特性　感性

$$H(j\omega)=\frac{Y(j\omega)}{Y_0}=\frac{\dfrac{\dot{I}}{\dot{U}_S}}{Y_0}=G+jB$$

【例】RLC串联电路, $u_S(t)=\sin(2\pi ft)$mV, 频率f=1MHz，调电容C，使电路发生谐振。I_0=100uA,U_{C0}=100mV。 试求:电路的R、L、C、Q及BW。

解：电压源的有效值：U_S=0.707mV

品质因数：$Q=\dfrac{U_{C0}}{U_S}=\dfrac{100}{0.707}=141$

带宽：

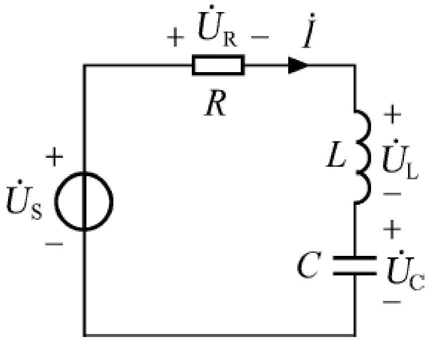
$$BW=\frac{f_0}{Q}=\frac{10^6}{141}=7.09\text{kHz}$$

$$R=\frac{U_S}{I_0}=\frac{0.707\times10^{-3}}{100\times10^{-6}}=7.07\Omega$$

$$L=\frac{1}{2\pi}\frac{R}{BW}=\frac{7.07}{6.28\times7.09\times10^3}=159\mu\text{H}$$

$$C=\frac{1}{\omega_0^2L}=\frac{1}{(6.28\times10^6)^2\times159\times10^{-6}}=159\text{pF}$$

二、RLC串联电路电压的频率特性



$$K_R(j\omega)=\frac{\dot{U}_R}{\dot{U}_S}=\frac{R}{R+j\left(\omega L-\dfrac{1}{\omega C}\right)}=\frac{1}{1+jQ\left(\dfrac{\omega}{\omega_0}-\dfrac{\omega_0}{\omega}\right)}$$

$$K_C(j\omega)=\frac{\dot{U}_C}{\dot{U}_S}=\frac{\dfrac{1}{j\omega C}}{R+j\left(\omega L-\dfrac{1}{\omega C}\right)}=\frac{-jQ\dfrac{\omega_0}{\omega}}{1+jQ\left(\dfrac{\omega}{\omega_0}-\dfrac{\omega_0}{\omega}\right)}$$

$$K_L(j\omega)=\frac{\dot{U}_L}{\dot{U}_S}=\frac{j\omega L}{R+j\left(\omega L-\dfrac{1}{\omega C}\right)}=\frac{jQ\dfrac{\omega_0}{\omega}}{1+jQ\left(\dfrac{\omega}{\omega_0}-\dfrac{\omega_0}{\omega}\right)}$$

二、 RLC串联电路电压的频率特性

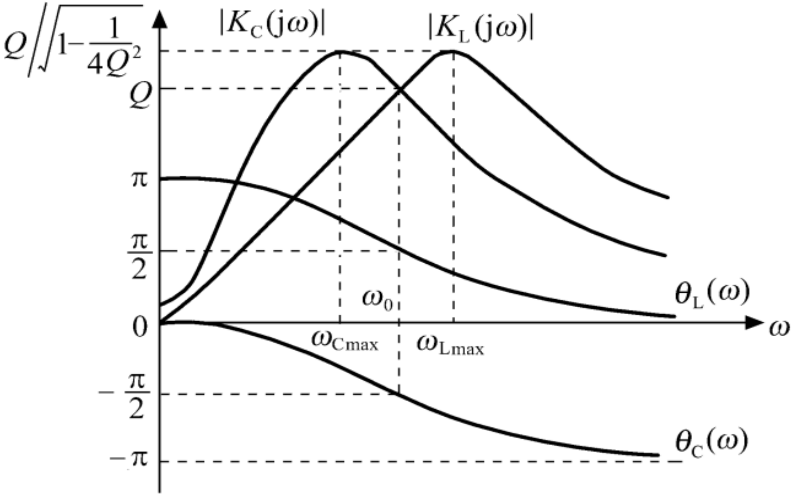


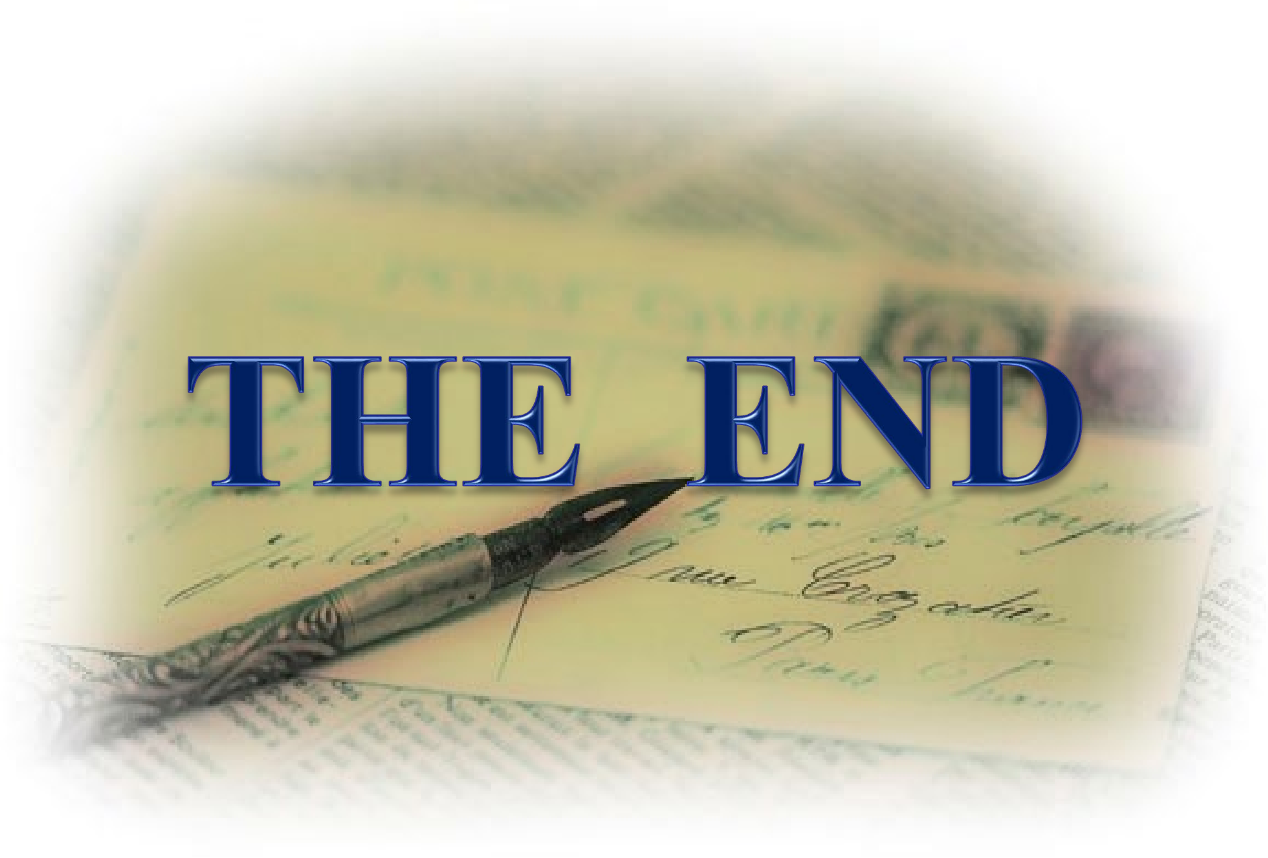
图 9-15 $K_C(j\omega)$、$K_L(j\omega)$ 频率特性

当Q≥10, 可以认为

$$\omega_{Cmax}=\omega_0=\omega_{Lmax}$$

即电压、电流都在ω_0达到最大

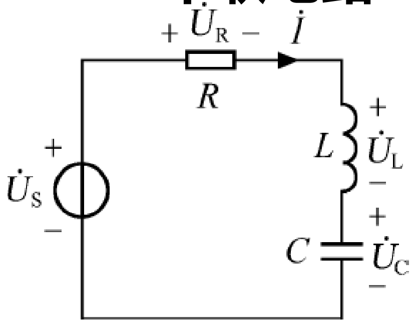
$$w_{C\max} = w_0\sqrt{1-\frac{1}{2Q^2}} \qquad w_{L\max} = w_0\Big/\sqrt{1-\frac{1}{2Q^2}}$$



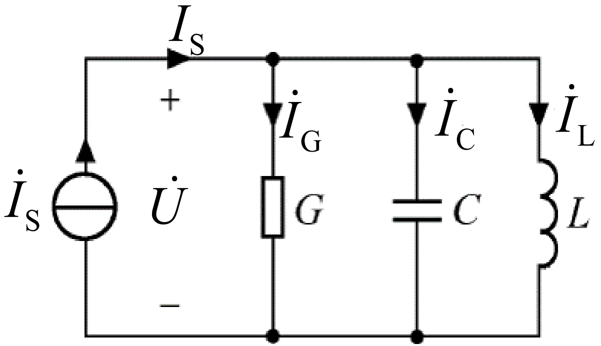
THE END

GCL并联谐振电路

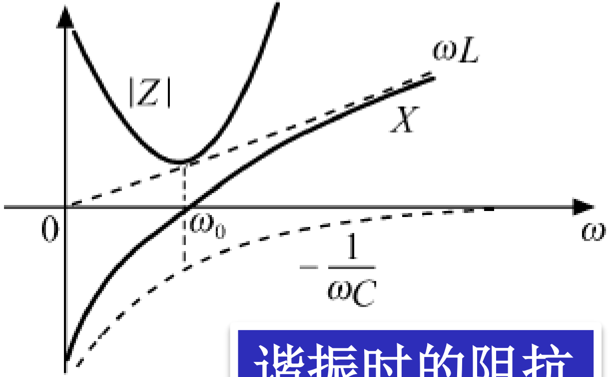
RLC串联电路



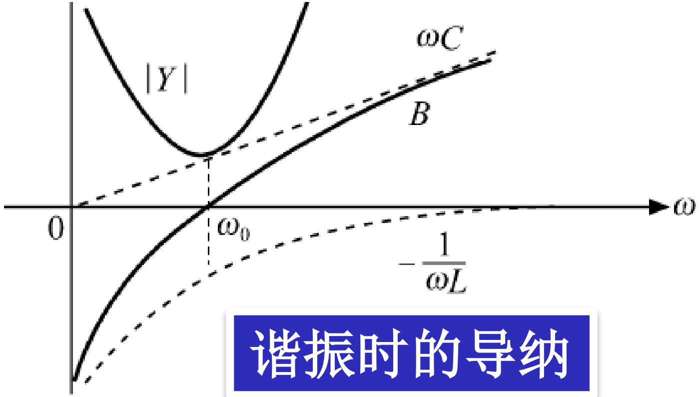
GCL并联电路



对偶电路



谐振时的阻抗



谐振时的导纳

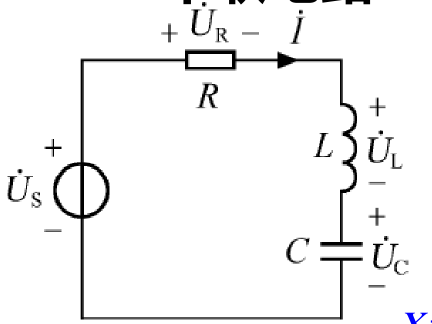
$$|Z(j\omega)|=R \qquad \theta_Z=0$$

$$X=\omega_0 L-\frac{1}{\omega_0 C}=0$$

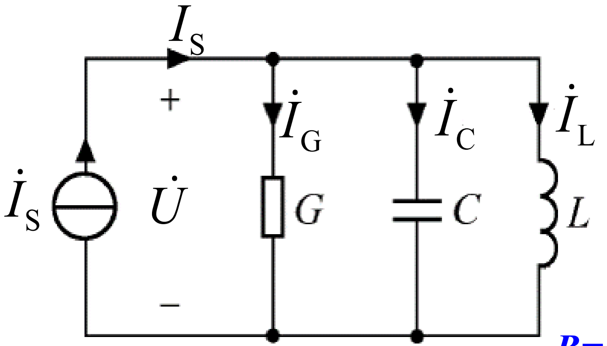
$$|Y(j\omega)|=G \qquad \theta_Y=-\theta_Z=0$$

$$B=\omega_0 C-\frac{1}{\omega_0 L}=0$$

RLC串联电路



GCL并联电路

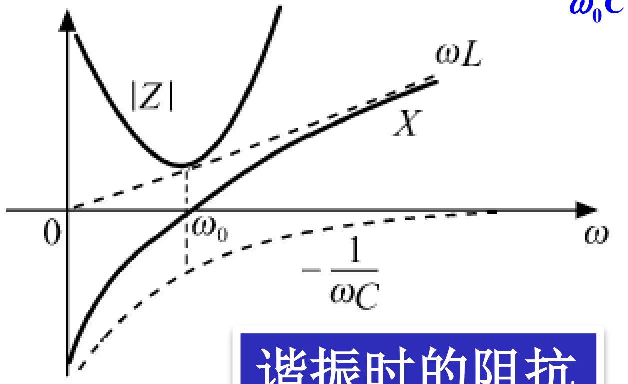


谐振频率

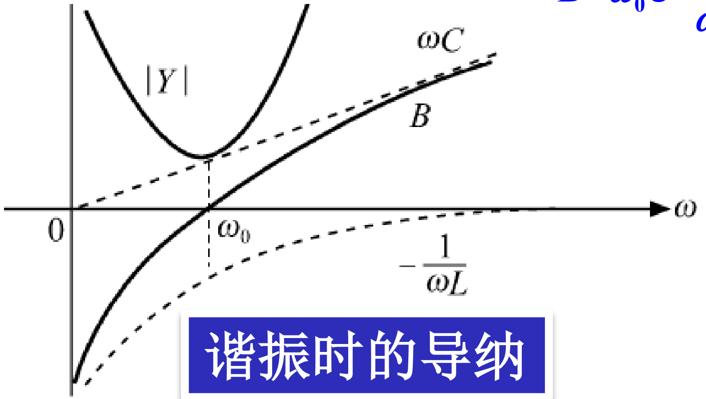
$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

$$X = \omega_0 L - \frac{1}{\omega_0 C} = 0$$

$$B = \omega_0 C - \frac{1}{\omega_0 L} = 0$$



谐振时的阻抗

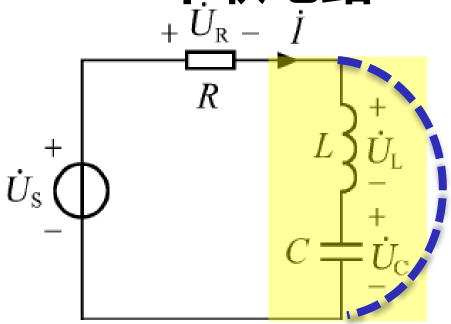


谐振时的导纳

$$|Z(\mathrm{j}\omega)| = R \quad \theta_Z = 0$$

$$|Y(\mathrm{j}\omega)| = G \quad \theta_Y = -\theta_Z = 0$$

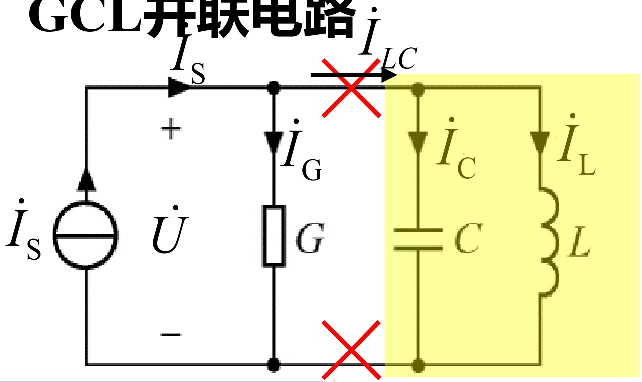
RLC串联电路



谐振频率

$$\omega=\omega_0=\frac{1}{\sqrt{LC}}$$

GCL并联电路



谐振时的电压

$$X=\omega_0 L-\frac{1}{\omega_0 C}=0$$

$$\dot{U}_{R0}=R\dot{I}_0=\dot{U}_S$$

$$\dot{U}_{L0}=\mathrm{j}\omega_0 L\dot{I}_0=\mathrm{j}\frac{\omega_0 L}{R}\dot{U}_S=\mathrm{j}Q\dot{U}_S$$

$$\dot{U}_{C0}=\frac{1}{\mathrm{j}\omega_0 C}\dot{I}_0=-\mathrm{j}\frac{1}{\omega_0 RC}\dot{U}_S=-\mathrm{j}Q\dot{U}_S$$

$$U_{L0}=U_{C0}=QU_S=QU_{R0}$$ **电压谐振**

$$Q=\frac{\omega_0 L}{R}=\frac{1}{\omega_0 CR}\qquad BW=\frac{\omega_0}{Q}$$

谐振时的电流

$$B=\omega_0 C-\frac{1}{\omega_0 L}=0$$

$$\dot{I}_{G0}=G\dot{U}_0=\dot{I}_S$$

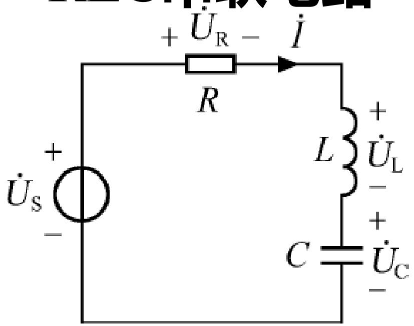
$$\dot{I}_{C0}=\mathrm{j}\omega_0 C\dot{U}_0=\mathrm{j}\frac{\omega_0 C}{G}\dot{I}_S=\mathrm{j}Q\dot{I}_S$$

$$\dot{I}_{L0}=\frac{1}{\mathrm{j}\omega_0 L}\dot{U}_0=-\mathrm{j}\frac{1}{\omega_0 LG}\dot{I}_S=-\mathrm{j}Q\dot{I}_S$$

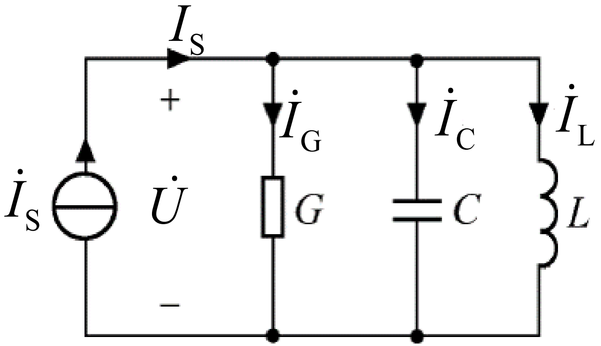
$$I_{L0}=I_{C0}=QI_S=QI_{R0}$$ **电流谐振**

$$Q=\frac{\omega_0 C}{G}=\frac{1}{\omega_0 LG}\qquad BW=\frac{\omega_0}{Q}$$

RLC串联电路



GCL并联电路



谐振频率

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

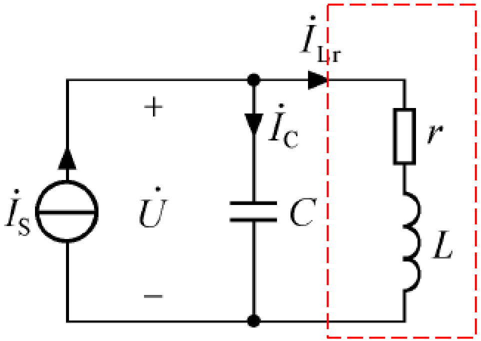
谐振时的能量

$$w = w_{L0} + w_{C0} = CU_{C0}^2 = LI_{L0}^2$$

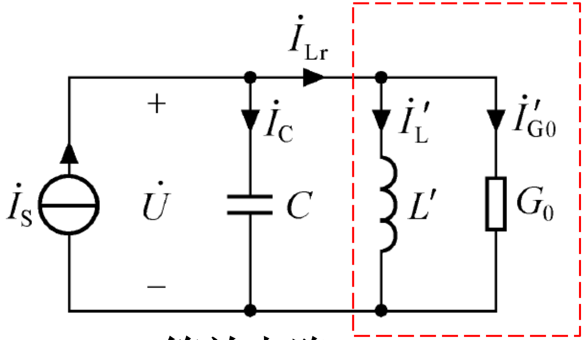
谐振时的频率特性

$$H(j\omega) = \frac{Y}{Y_0} = \frac{1}{1 + jQ\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)}$$

$$H(j\omega) = \frac{Z}{Z_0} = \frac{1}{1 + jQ\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)}$$



实际电路



等效电路

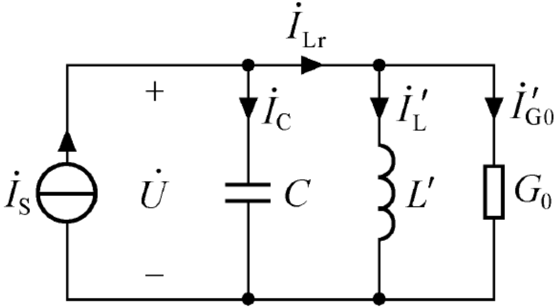
实际并联谐振电路

$$Z(j\omega)=\frac{(r+j\omega L)\dfrac{1}{j\omega C}}{r+j\omega L+\dfrac{1}{j\omega C}}\qquad r<<\omega_0 L$$

$$Z(j\omega)=\frac{L/C}{r+j(\omega L-\dfrac{1}{\omega C})}=\frac{1}{\dfrac{C}{L}r+j(\omega C-\dfrac{1}{\omega L})}$$

$$Y(j\omega)=\frac{C}{L}r+j(\omega C-\frac{1}{\omega L})=G_0+jB$$

$$G_0=\frac{C}{L}r,L'=L\qquad \omega=\omega_0=\frac{1}{\sqrt{LC}}$$



谐振导纳 $Y_0 = G_0 = \dfrac{C}{L} r$

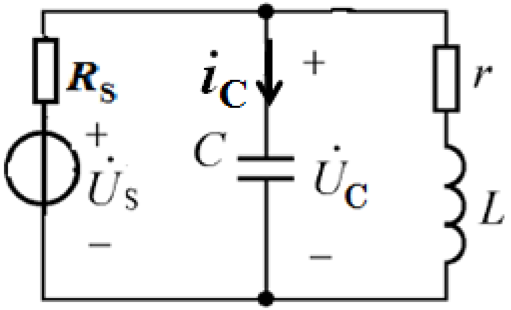
$$Y(\mathrm{j}\omega) = \frac{C}{L} r + \mathrm{j}(\omega L - \frac{1}{\omega C}) = G_0 + \mathrm{j}B$$

谐振阻抗 $Z_0 = R_0 = \dfrac{1}{Y_0} = \dfrac{1}{G_0} = \dfrac{L}{C \cdot r}$

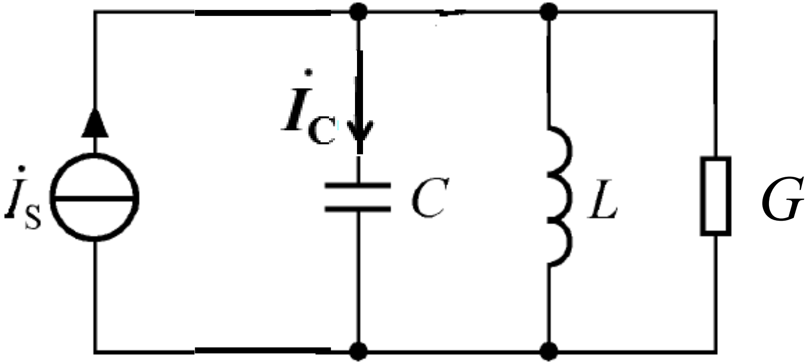
实际的并联谐振电路的品质因数为

$$Q = \frac{\omega_0 C}{G_0} = \frac{1}{\omega_0 L G_0}$$

【例1】 已知U_S=6V， R_S=30kΩ, L=54μH, C=100pF, r =9 Ω， 电路发生并联谐振。试求：谐振角频率ω_0、电容电流I_C和整个电路品质因数Q。



解：等效变换， G_S和G_0合并



$$G=G_S+G_0=\frac{1}{R_S}+\frac{Cr}{L}=5\times10^{-5}\text{S}$$

$$\omega_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{54\times10^{-6}\cdot10^{-10}}}$$

$$=1.36\times10^{7}\ \text{rad/s}$$

$$Q=\frac{\omega_0 C}{G}=27.2$$

$$I_C=QI_S=27.2\times0.2=5.44\text{mA}$$

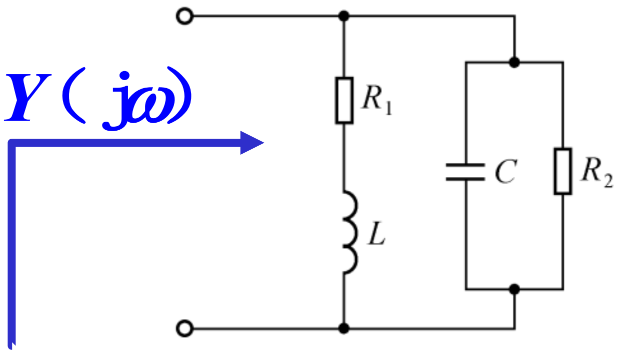
一般谐振电路

多个电抗元件组成的谐振电路

一般情况：

串联谐振:策动点阻抗虚部为零

并联谐振:策动点导纳虚部为零

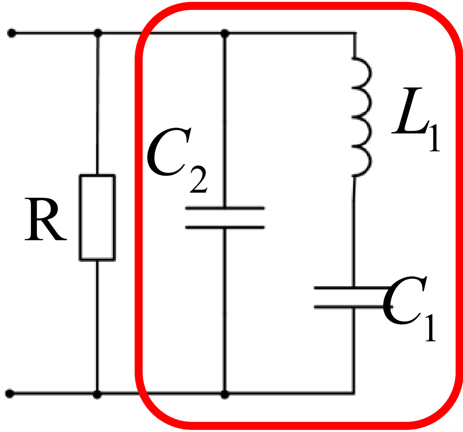


$$\mathrm{Im}[Y(j\omega)]=0$$

特殊情况：条件：电路中全部电抗元件组成纯电抗局部电路（支路）发生谐振。

1.局部电路的阻抗（电抗）为零时，该局部电路发生串联谐振。

2.局部电路的导纳（感纳）为零时，该局部电路发生并联谐振。



【例 10-6】 在如图 10-20（a）所示的电路中，已知 $R_1=50\Omega$，$R_2=100\Omega$，$C=10\mu F$，电路在 $u_S(t)=\cos(1000t)$ V 激励下发生谐振。试求电感 L 和电流 $i(t)$。

（a） （b）

策动点阻抗： $Z=R_1+j\omega L+R_2//\dfrac{1}{j\omega C}=50+j1000L+50-j50$

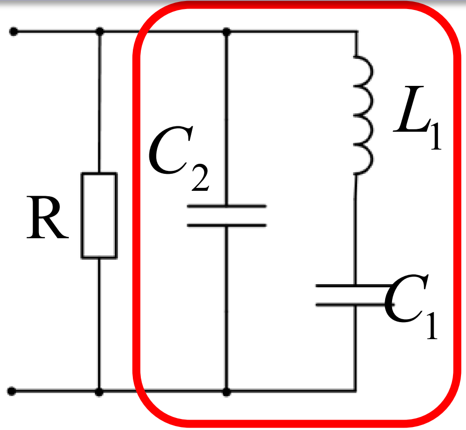
$$=100+j(1000L-50)$$

电路发生谐振，策动点阻抗虚部为0： $1000L-50=0$ $L=50\text{mH}$

谐振电流 $\dot{I}_0=\dfrac{\dot{U}_S}{Z_0}=\dfrac{0.707\angle 0^\circ}{100}=7.07\angle 0^\circ\text{mA}$

$$i(t)=10\cos(1000t)\text{mA}$$

【例】 求右图所示电路的串、并联谐振频率。

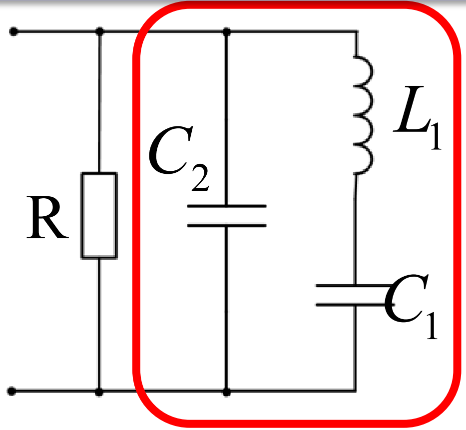


$$Z=\frac{1}{j\omega C_2}//(\frac{1}{j\omega C_1}+j\omega L_1)=\frac{\frac{1}{j\omega C_2}\times(\frac{1}{j\omega C_1}+j\omega L_1)}{\frac{1}{j\omega C_2}+\frac{1}{j\omega C_1}+j\omega L_1}$$

$$=\frac{-\frac{1}{\omega C_2}\times j(\omega L_1-\frac{1}{\omega C_1})}{(\omega L_1-\frac{1}{\omega C_2}-\frac{1}{\omega C_1})}$$

串联谐振：$Z=0 \quad \omega_{01}=\frac{1}{\sqrt{C_1L_1}}$

【例】 求右图所示电路的串、并联谐振频率。

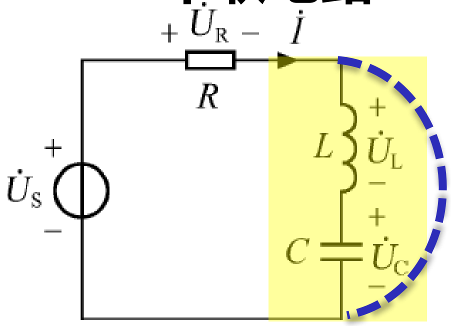


$$Y = \mathrm{j}\omega C_2 + \frac{1}{\frac{1}{\mathrm{j}\omega C_1} + \mathrm{j}\omega L_1} = \frac{\mathrm{j}\omega(C_1 + C_2 - \omega^2 C_1 C_2 L_1)}{1 - \omega^2 C_1 L_1}$$

并联谐振：$Y = 0$

$$\omega_{02} = \frac{1}{\sqrt{L_1 \dfrac{C_1 C_2}{C_1 + C_2}}}$$

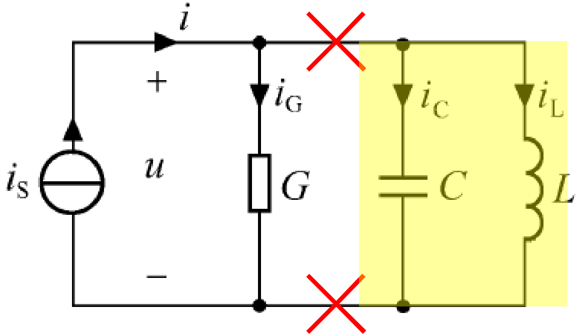
RLC串联电路



谐振频率

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

GCL并联电路



纯电抗元件部分
相当于对外电路
相当于短路

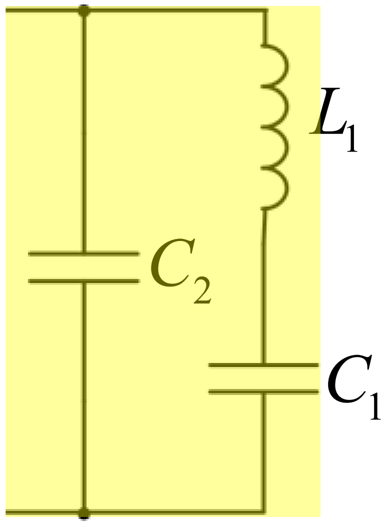
纯电抗元件部分
相当于对外电路
相当于开路

另解：**串联谐振**，多个电抗元件部分相当于对外电路相当于短路

串联谐振频率 $\omega_0=\dfrac{1}{\sqrt{L_1C_1}}$

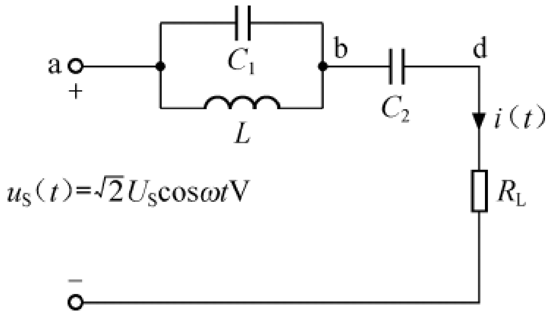
并联谐振，对外电路相当于开路

并联谐振频率 $\omega_0=\dfrac{1}{\sqrt{L_1C_{\text{eq}}}}=\dfrac{1}{\sqrt{L_1\dfrac{C_1C_2}{C_1+C_2}}}$



【例】L=10mH, 试求C_1和C_2，使电源频率为100kHz时流经R_L的电流为0，而电源频率为50kHz时流经R_L的电流为最大。

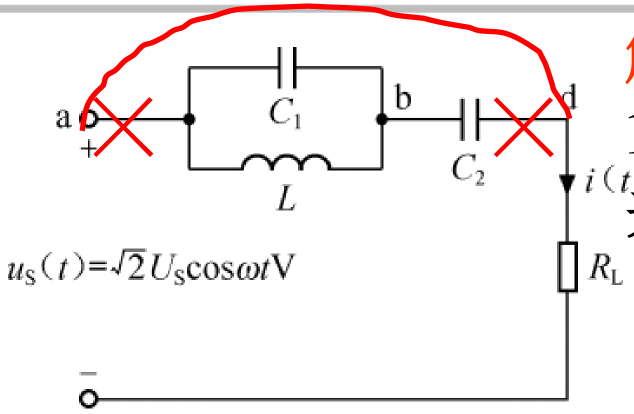
解1：ad间纯电抗，对100kHz($\omega_0=2\pi\times10^5$)相当于开路（发生并联谐振），$Y_{ab}=0$



$$j\omega_0 C_1+\frac{1}{j\omega_0 L}=0 \qquad C_1=253\text{pF}$$

ad间对 50kHz($0.5\omega_0$)相当于短路（串联谐振），$Z_{ad}=0$

$$\frac{1}{j0.5\omega_0 C_2}+\frac{\frac{1}{j0.5\omega_0 C_1}\cdot j0.5\omega_0 L}{\frac{1}{j0.5\omega_0 C_1}+j0.5\omega_0 L}=0 \qquad C_2=746\text{pF}$$



解2：ad间纯电抗，对100kHz(ω_0=2π×10^5)相当于开路（发生并联谐振）

$$\omega_0=\frac{1}{\sqrt{LC_1}}$$

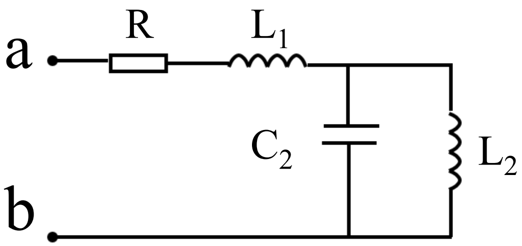
C_1 =253pF

ad间对 50kHz(0.5ω_0)相当于短路（串联谐振）

$$\omega_0=\frac{1}{\sqrt{L(C_1+C_2)}}$$

C_2=746pF

例：求电路发生串、并联谐振的谐振频率



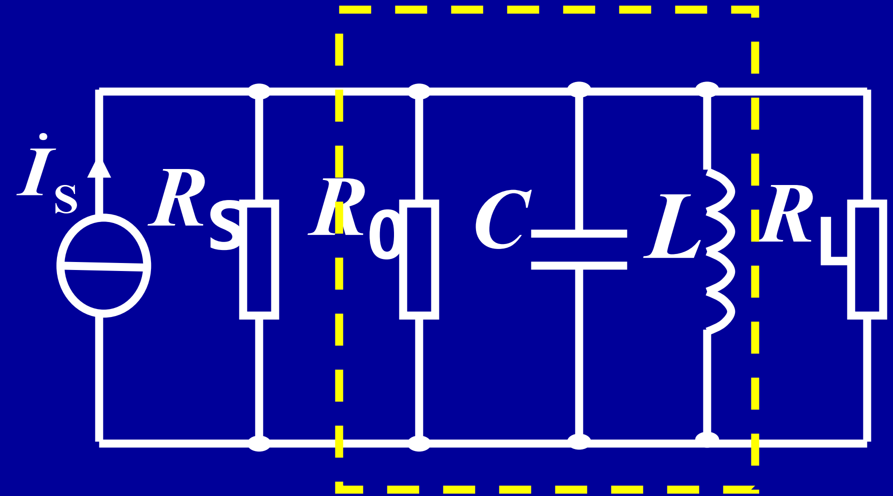
串联谐振 $$\omega_{01}=\frac{1}{\sqrt{C_2(L_1//L_2)}}$$

并联谐振 $$\omega_{02}=\frac{1}{\sqrt{C_2L_2}}$$

9-6 电源内阻及负载电阻对谐振的影响

加载回路

$$G_S=\frac{1}{R_S},G_L=\frac{1}{R_L},G_0=\frac{1}{R_0}$$



$$\omega_0=\frac{1}{\sqrt{LC}}$$

总电导为： $G_0'=G_S+G_L+G_0$

谐振阻抗： $Z_0'=R_0'=\dfrac{1}{G_0'}$

空载时品质因数： $$Q = \frac{1}{G_0}\sqrt{\frac{C}{L}} = \frac{\sqrt{C/L}}{G_0}$$

加载后品质因数：

$$Q' = \frac{1}{G_0'}\sqrt{\frac{C}{L}} = \frac{\sqrt{C/L}}{G_S + G_L + G_0}$$

$$= \frac{1}{1 + \frac{G_S}{G_0} + \frac{G_L}{G_0}} \cdot Q << Q$$

结论：加载后，品质因数比空载时下降，选择性变差，通频带变宽。

可见，为了不使品质因数下降太多，并联谐振电路希望与高内阻电源相接。